A microscopic approach to nuclear dynamics

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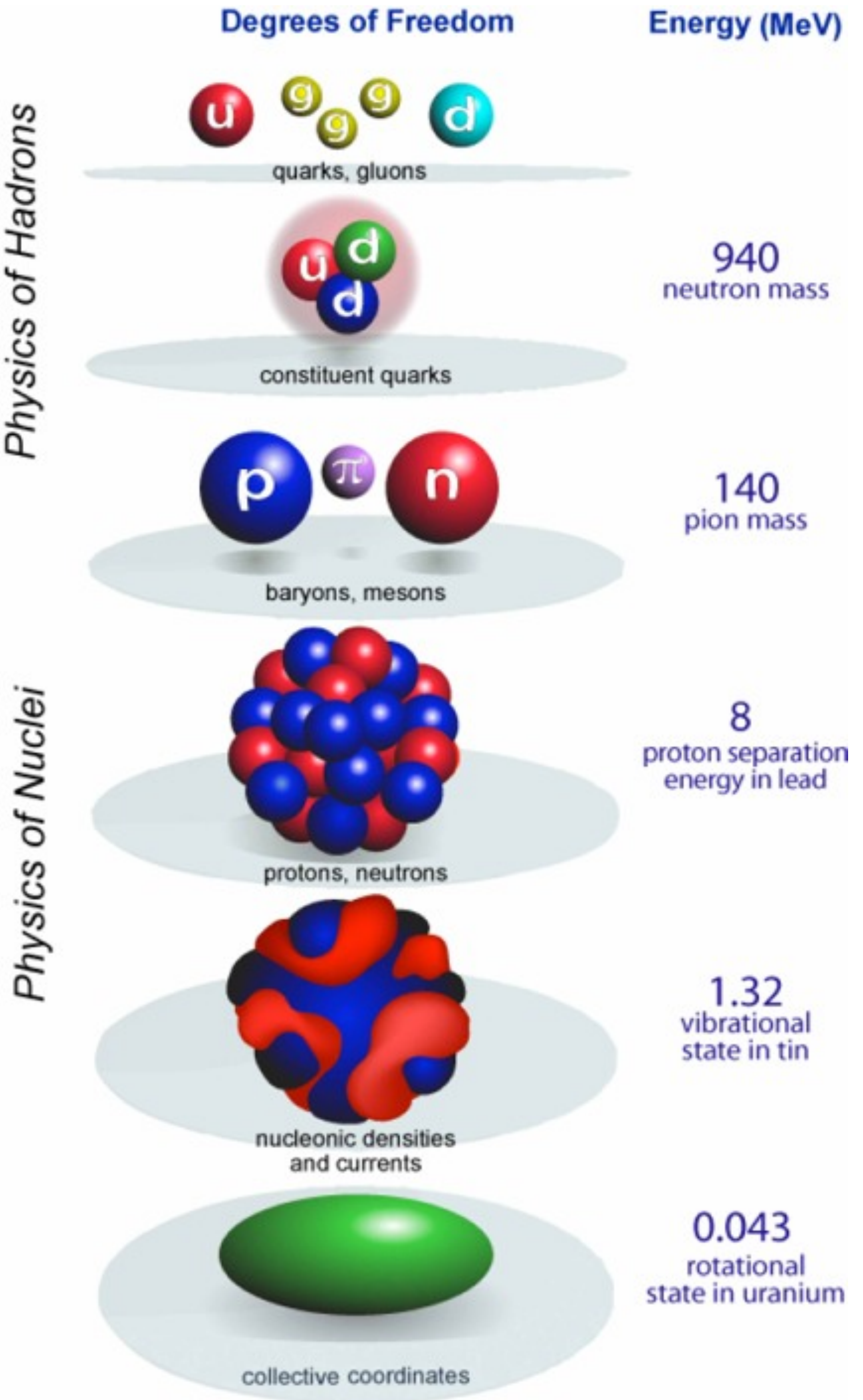
Introduction

- Quantum dynamics of complex systems (**nuclei**, molecules, BEC, atomic clusters...)
- Collectivity: from **vibrations** to **collisions**
- Interplay with **single-particle** d.o.f. (Giant Resonance decay, Competition fusion/ transfer...)
- Quantum many-body problem

Microscopic, up to which scale?



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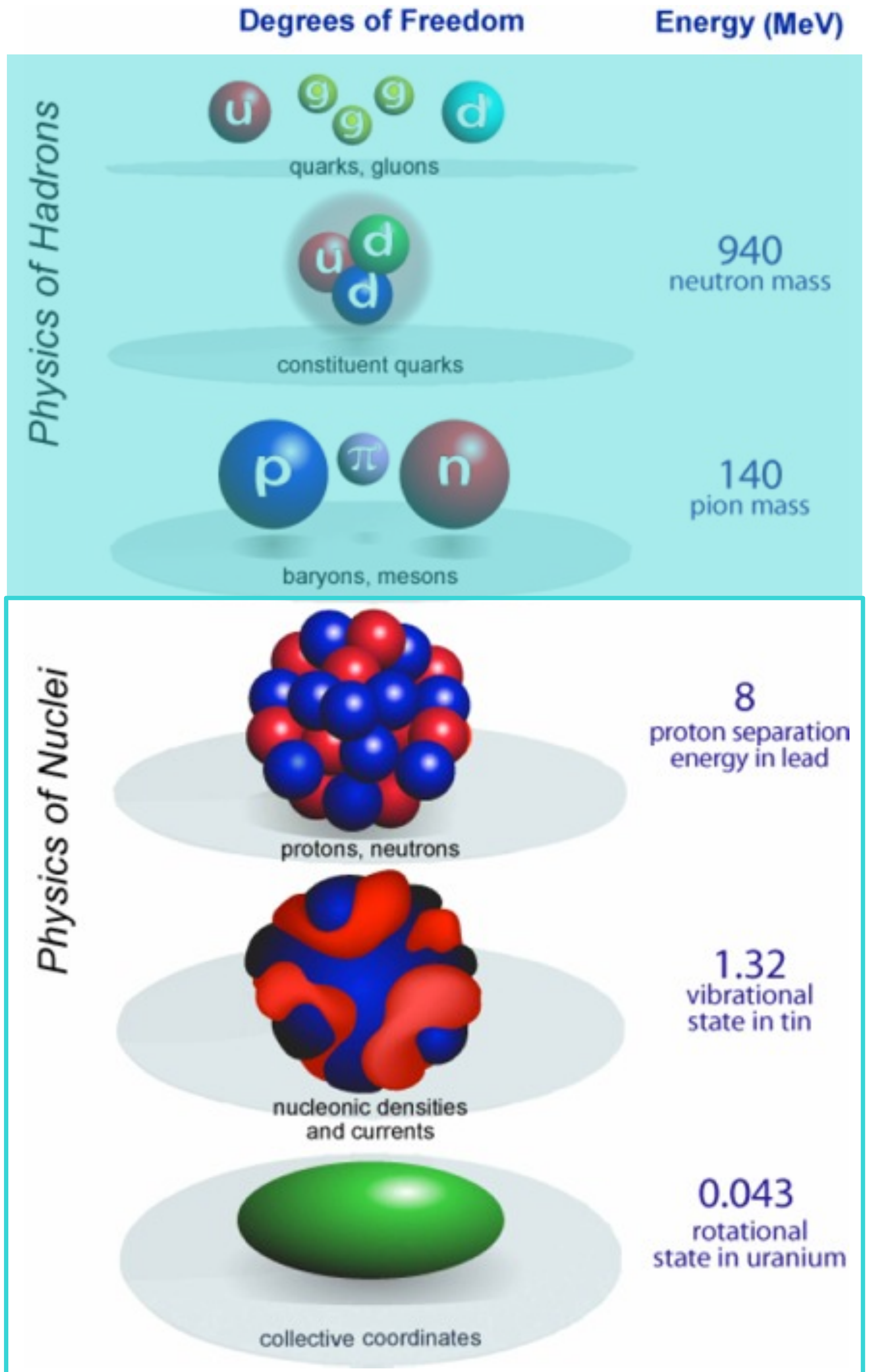
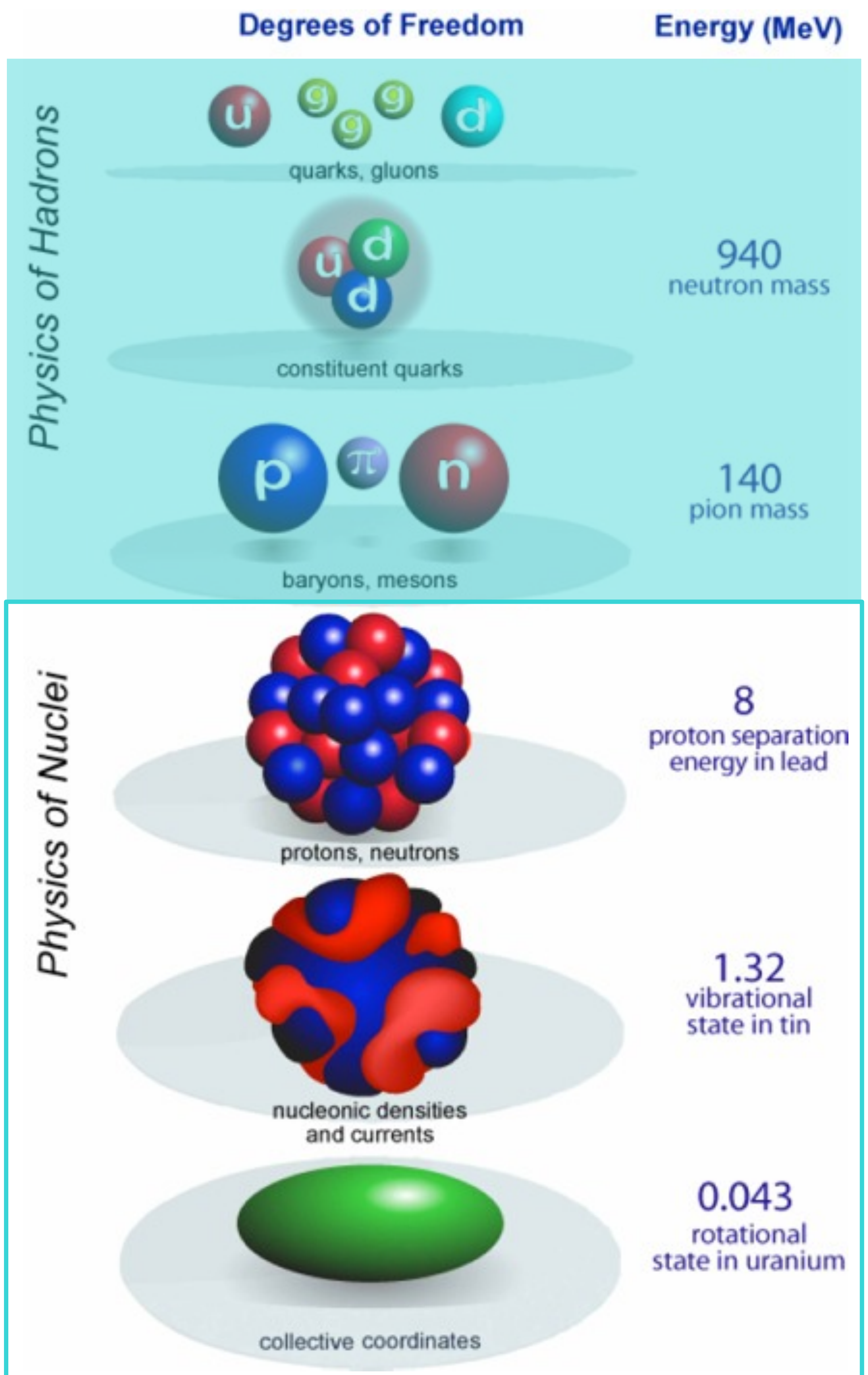


Microscopic, up to which scale?

Ingredients of the model:

- nucleon-nucleon interaction
- approximations of the many-body problem



Hamiltonian

$$\hat{H} = \hat{T} + \hat{V}$$

$$\hat{T} = \sum_{i=1}^{N} \frac{\hat{p}^2_{(i)}}{2m}$$

$$\hat{V} = \frac{1}{2} \sum_{i \neq j=1}^{N} \hat{v}(i, j)$$

Hamiltonian

Separation into a mean field U and a residual interaction

$$\hat{V} = \hat{U} + \hat{V}_{res} \qquad \text{with} \qquad \hat{U} = \sum_{i=1}^{N} \hat{u}(i)$$

Mean-Field approximation: neglect V_{res}

Each nucleon is assumed to evolve independently in the MF generated by the other nucleons. The interactions are «averaged» into a MF.

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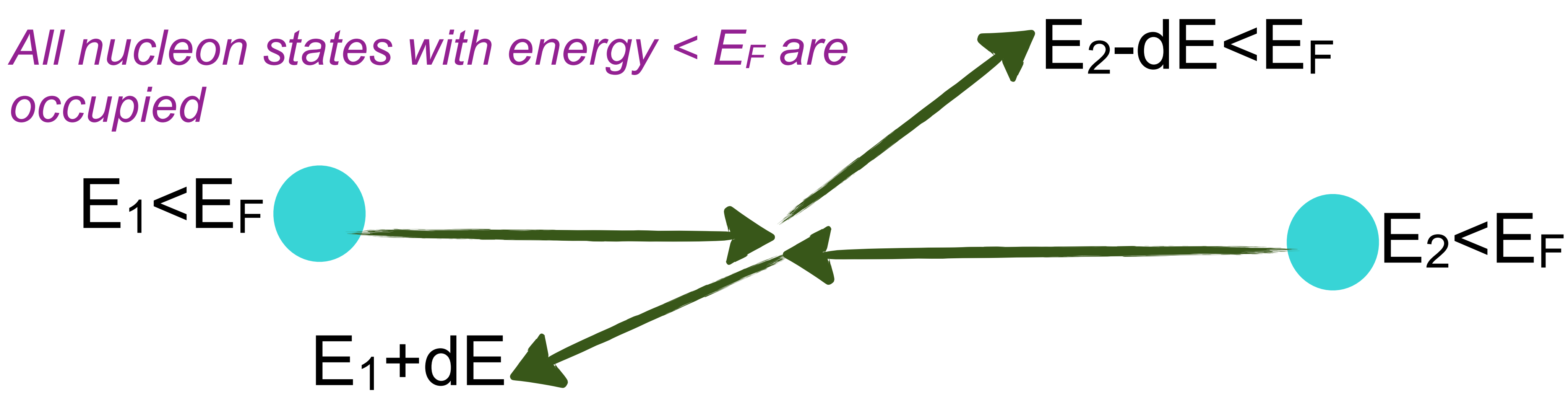

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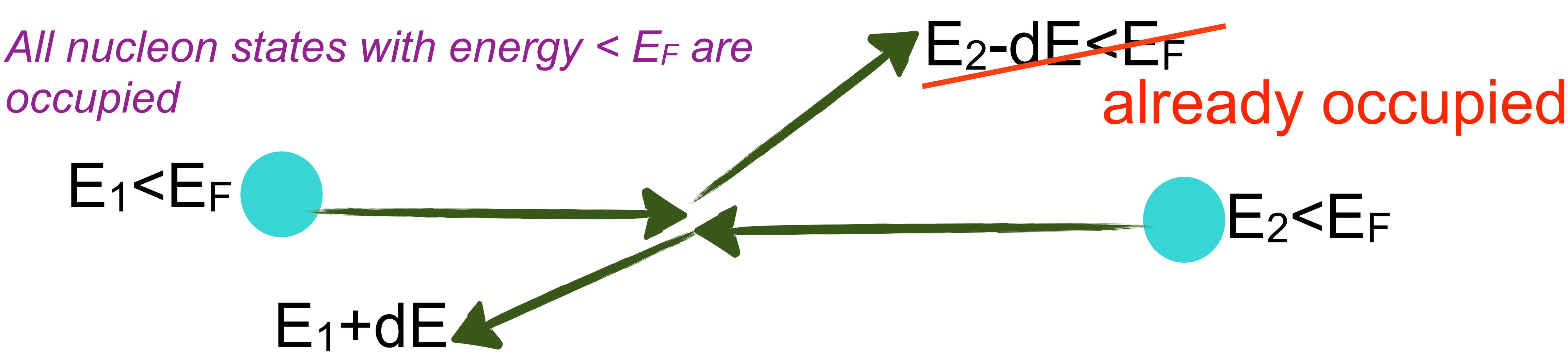

Hartree-Fock

- Mean-field determined from the interaction

$$\hat{u}(i) = \sum_{j=1}^{N} \langle \varphi_j | \hat{\bar{v}}(i,j) | \varphi_j \rangle$$

- Self-consistent mean-field

$$\hat{u} \equiv \hat{u}[\varphi_1, \varphi_2, \cdots \varphi_N] \equiv \hat{u}[\rho]$$

- HF equation

$$\left(\frac{\hat{p}^2}{2m} + \hat{u}[\rho] \right) |\varphi_i\rangle \equiv \hat{h}[\rho] |\varphi_i\rangle = e_i |\varphi_i\rangle \quad \text{for } i = 1, 2 \cdots N$$

Practical aspects of HF calculations

Imaginary time method

for a ground state with no self-consistency ($\hat{h} \neq \hat{h}[\rho]$ *):*

$$|\varphi_{init}\rangle = \sum_i C_i |\varphi_i\rangle$$

$$e^{-i\hat{h}t} \rightarrow e^{-\beta\hat{h}}|\varphi_{init}\rangle = \sum_i C_i e^{-\beta E_i}|\varphi_i\rangle \xrightarrow[\beta\to\infty]{} C_0 e^{-\beta E_0}|\varphi_0\rangle$$

Practical aspects of HF calculations

HF calculations

Start with Nilsson or harmonic oscillator w.f.

Imaginary time method

***for a N lowest states of h[ρ]:* iterative process (evolution with $\Delta\beta$)**

$$\{|\varphi_1^{[n]}\rangle \cdots |\varphi_N^{[n]}\rangle\} \quad \Rightarrow \quad \rho^{[n]} \quad \Rightarrow \hat{h}^{[n+1]} = \hat{h}[\rho^{[n]}]$$

$$\Uparrow \qquad\qquad\qquad\qquad \Downarrow$$

$$|\varphi_i^{[n+1]}\rangle = \frac{1}{N_i}\left(|\varphi_i'\rangle - \sum_{j=0}^{i-1} \langle\varphi_j^{[n+1]}|\varphi_i'\rangle \; |\varphi_j^{[n+1]}\rangle\right) \quad \Leftarrow \quad |\varphi_i'\rangle = (1 - \Delta\beta\,\hat{h}^{[n+1]}) \; |\varphi_i^{[n]}\rangle$$

(Graham-Schmidt orthonormalization)

Practical aspects of HF calculations

HF calculations

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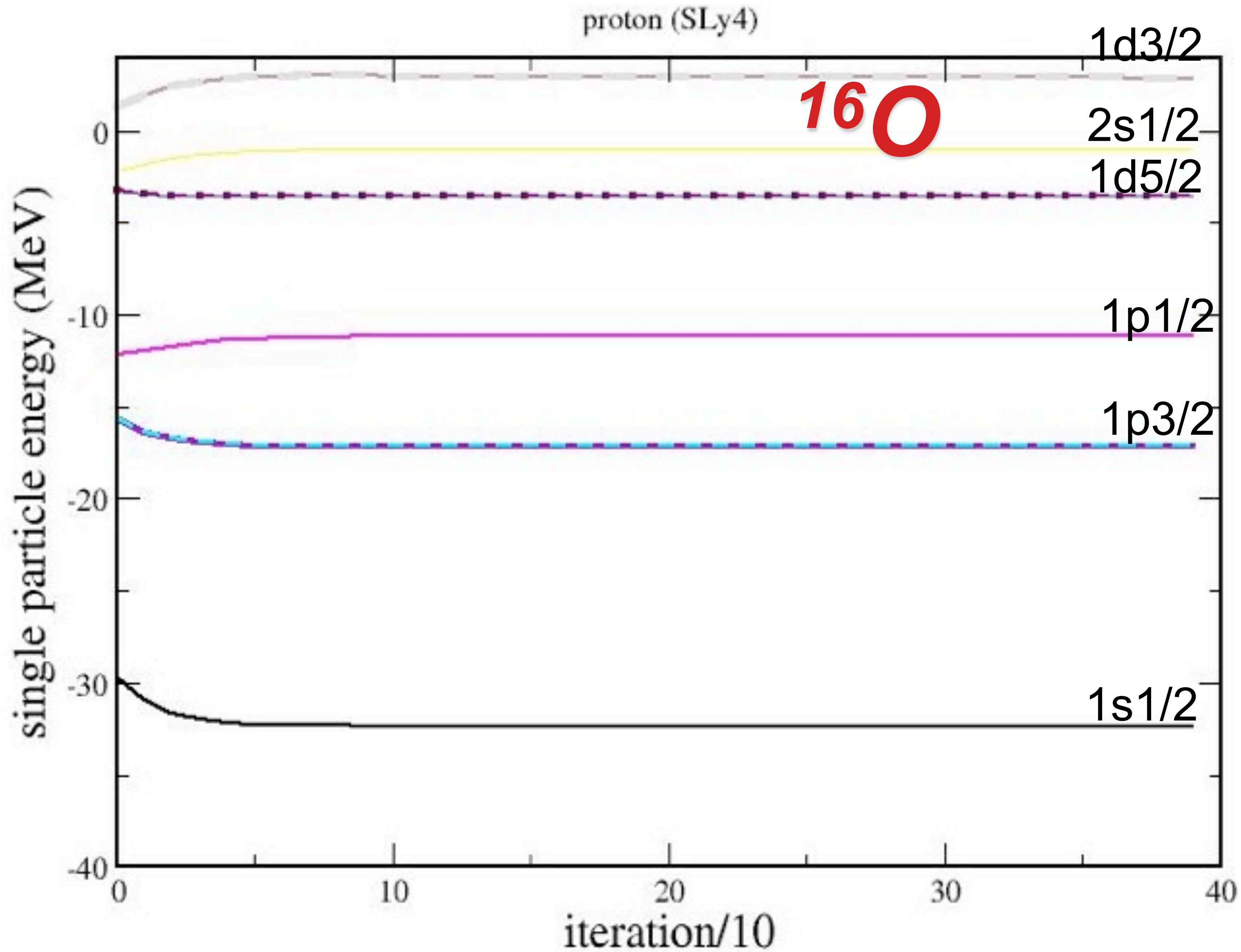


ev8, P. Bonche et al., Comp. Phys. Com. 171, 49 (2005)

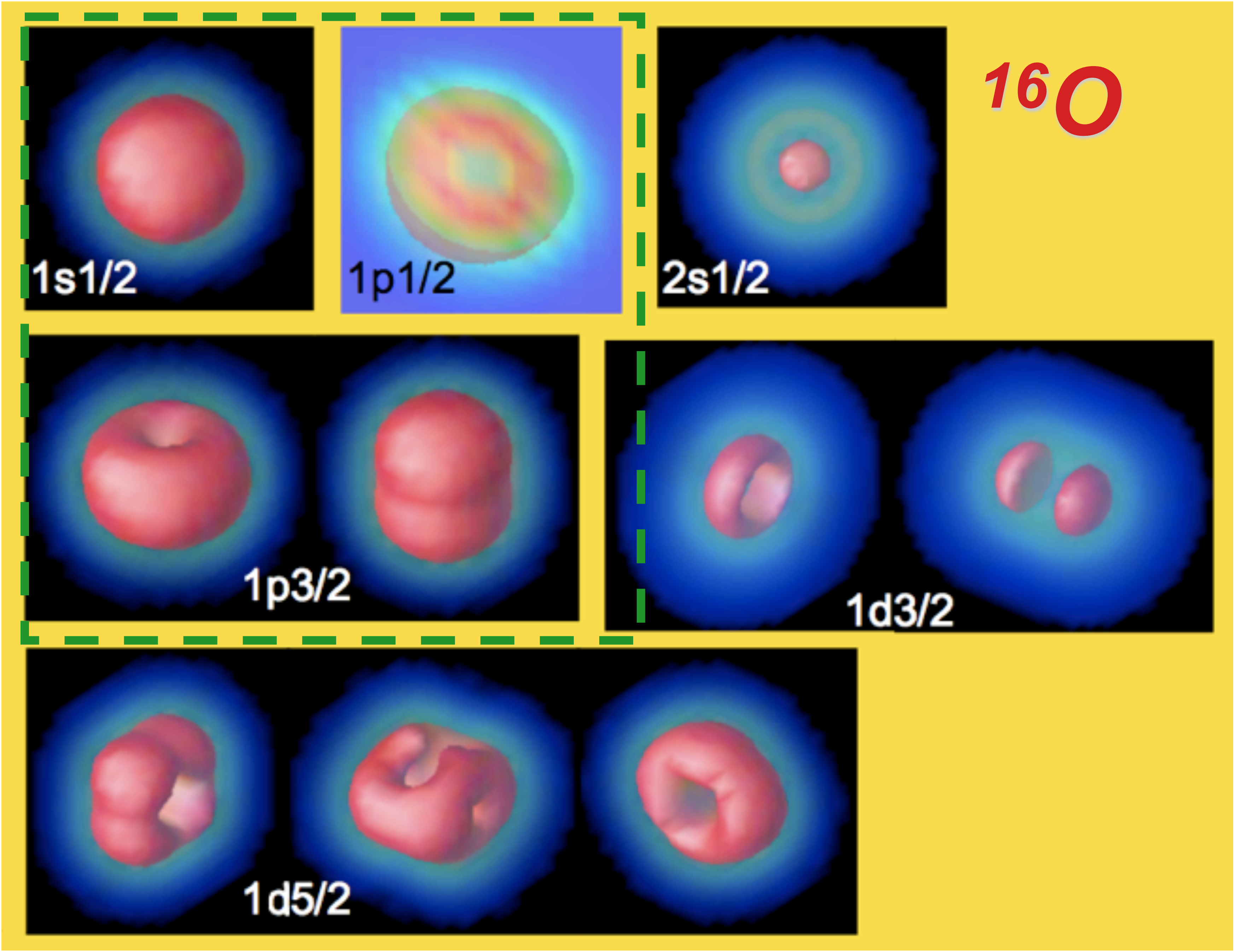
Practical aspects of HF calculations

HF calculations

Start with Nilsson w.f.

Imaginary time method

Occupied neutron w.f.



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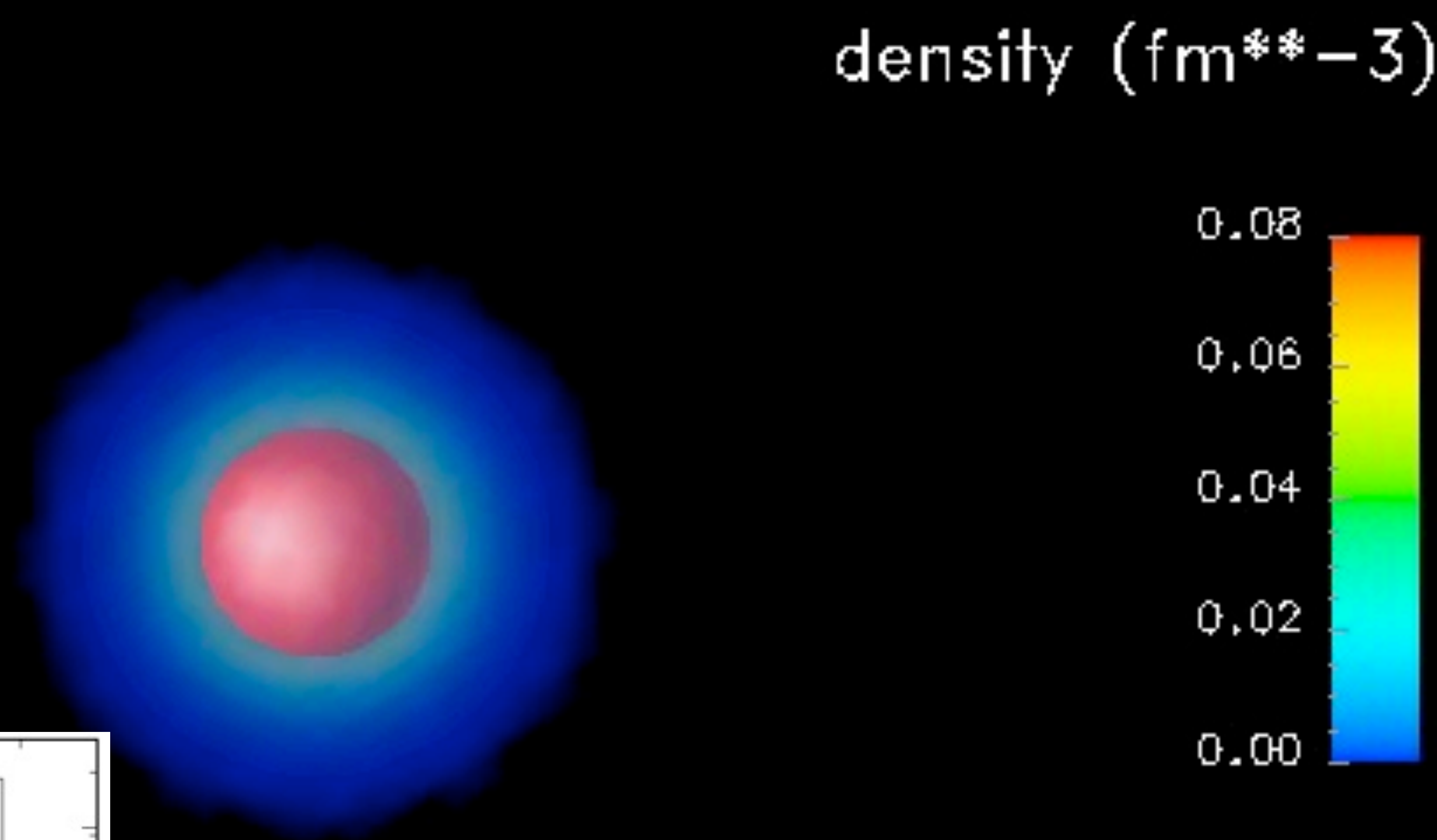
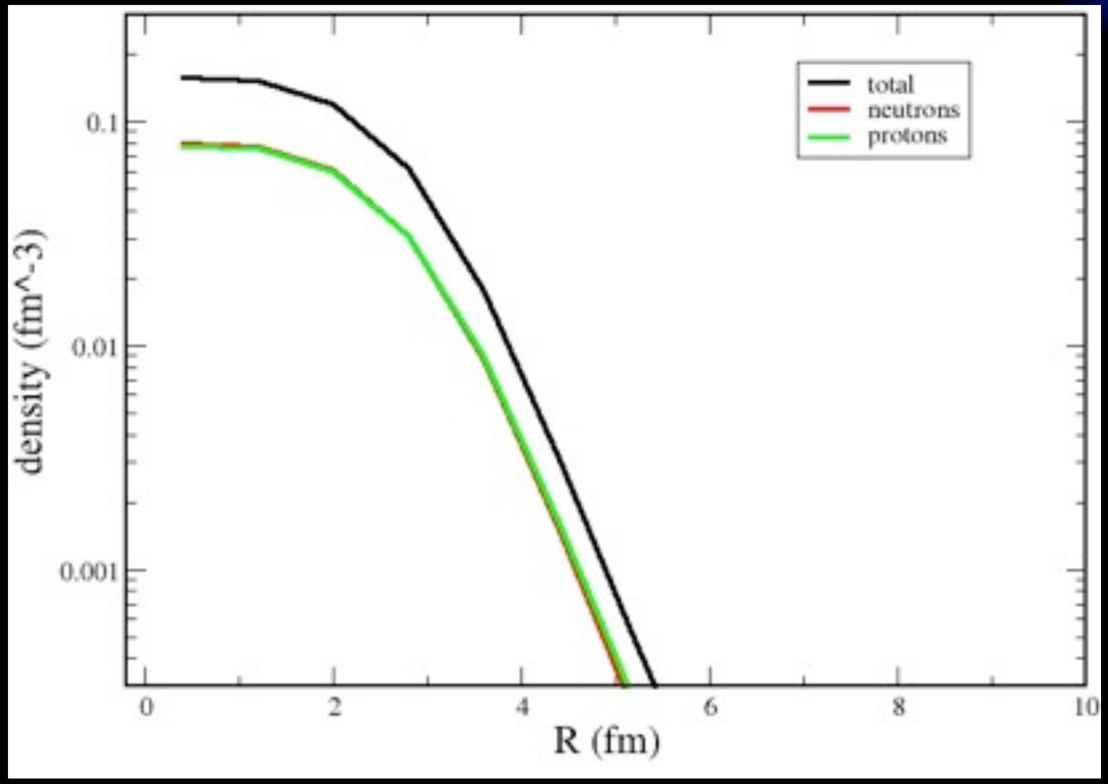
Ground-state density from HF calculations

^{16}O





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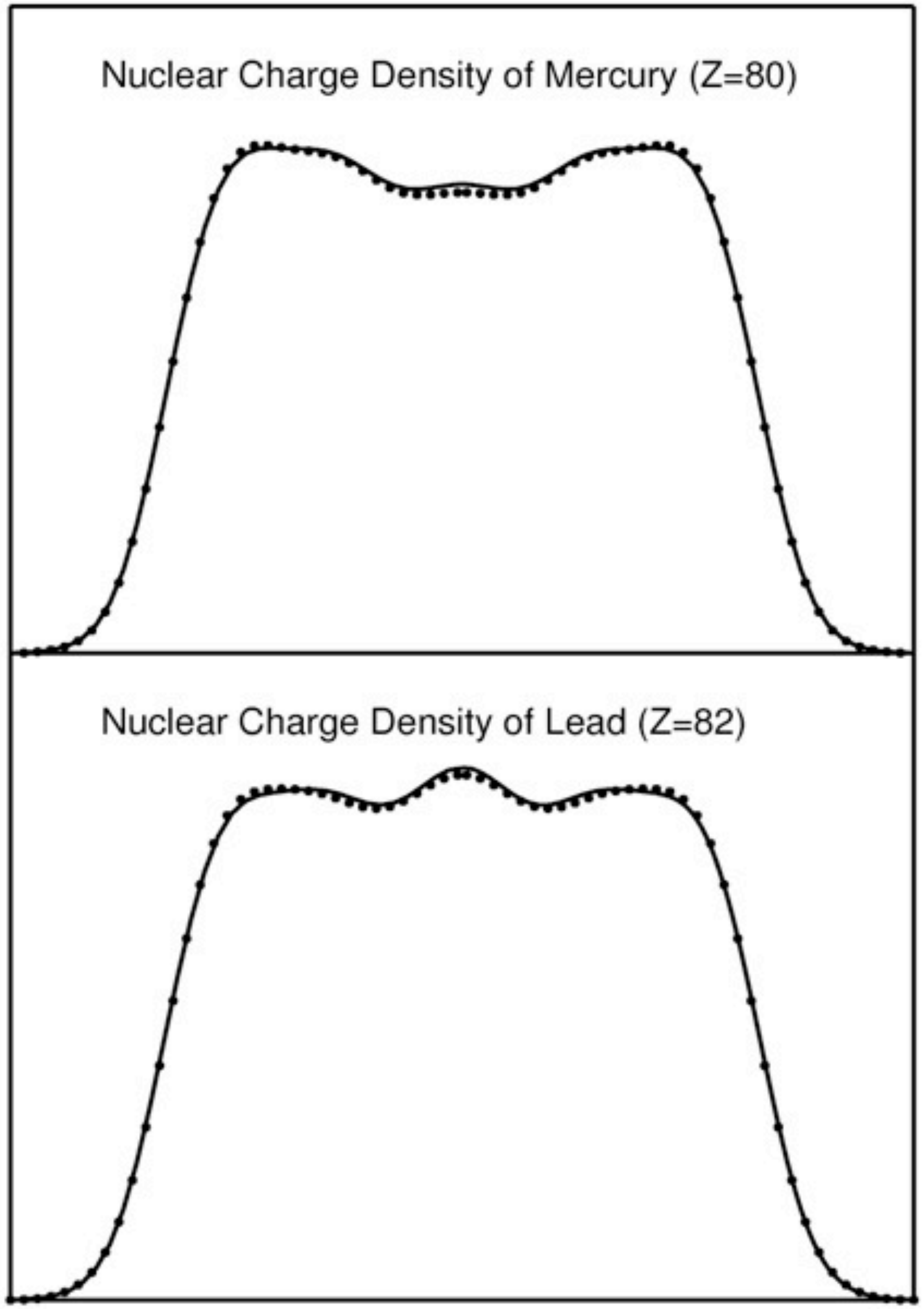

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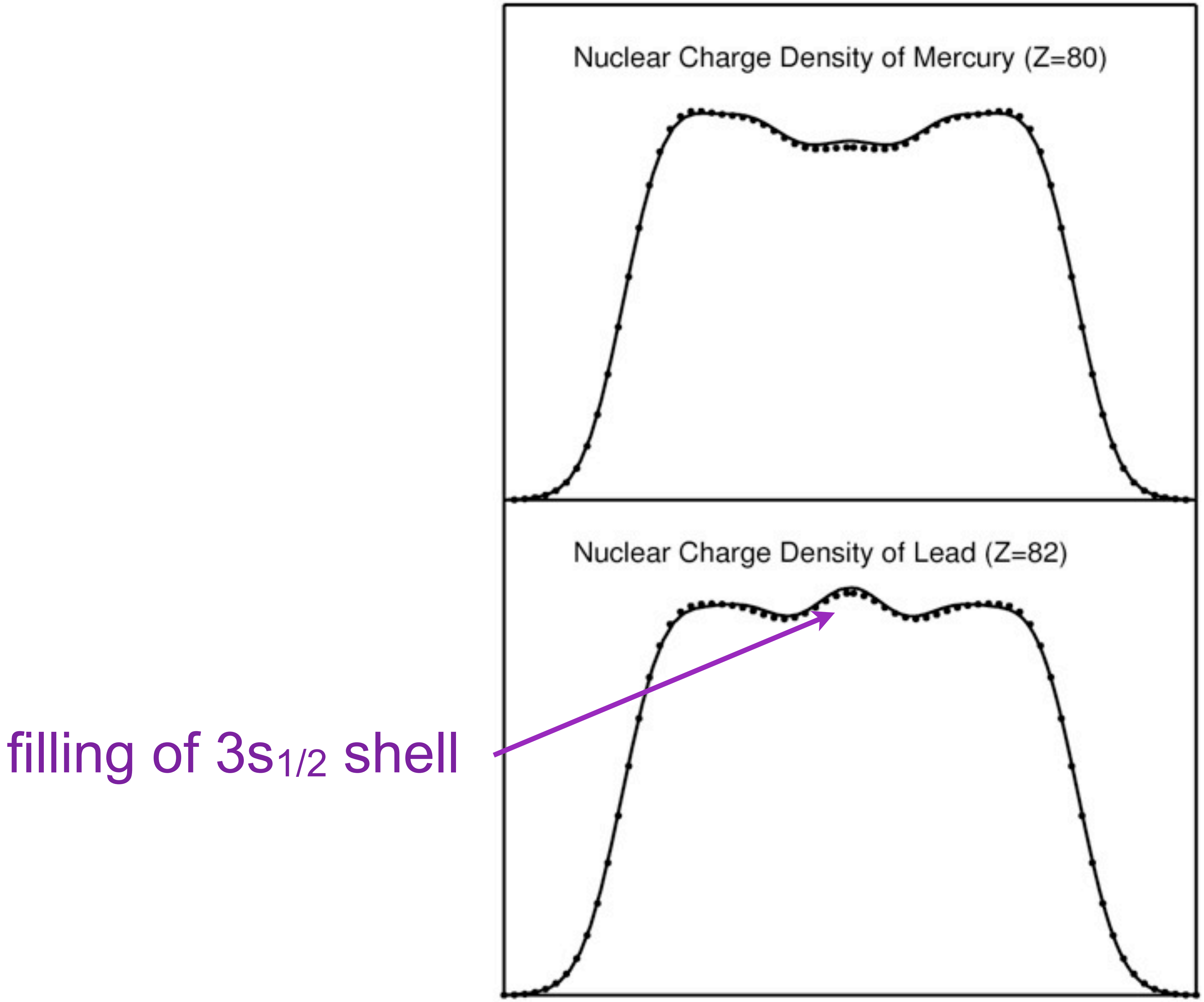
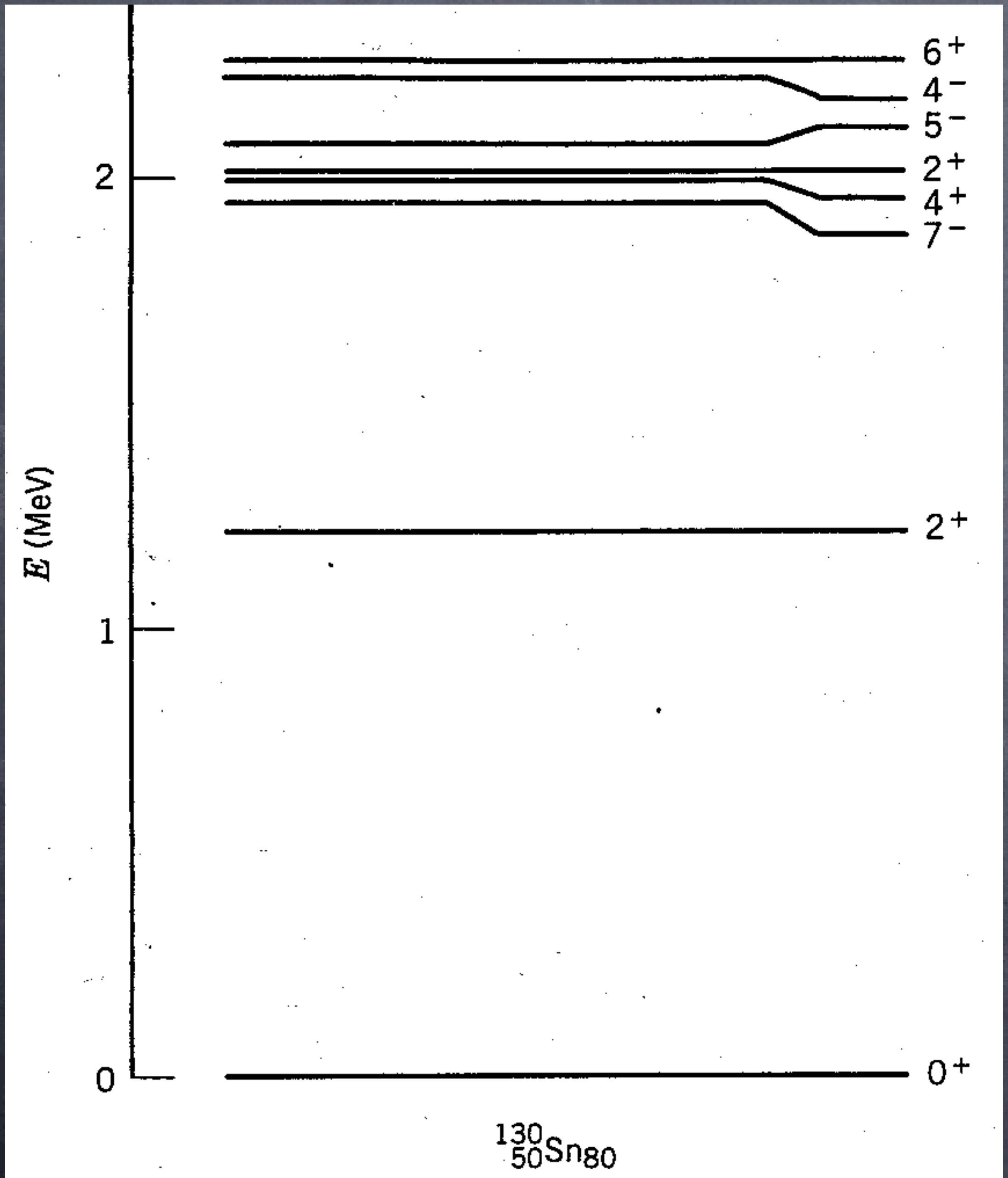
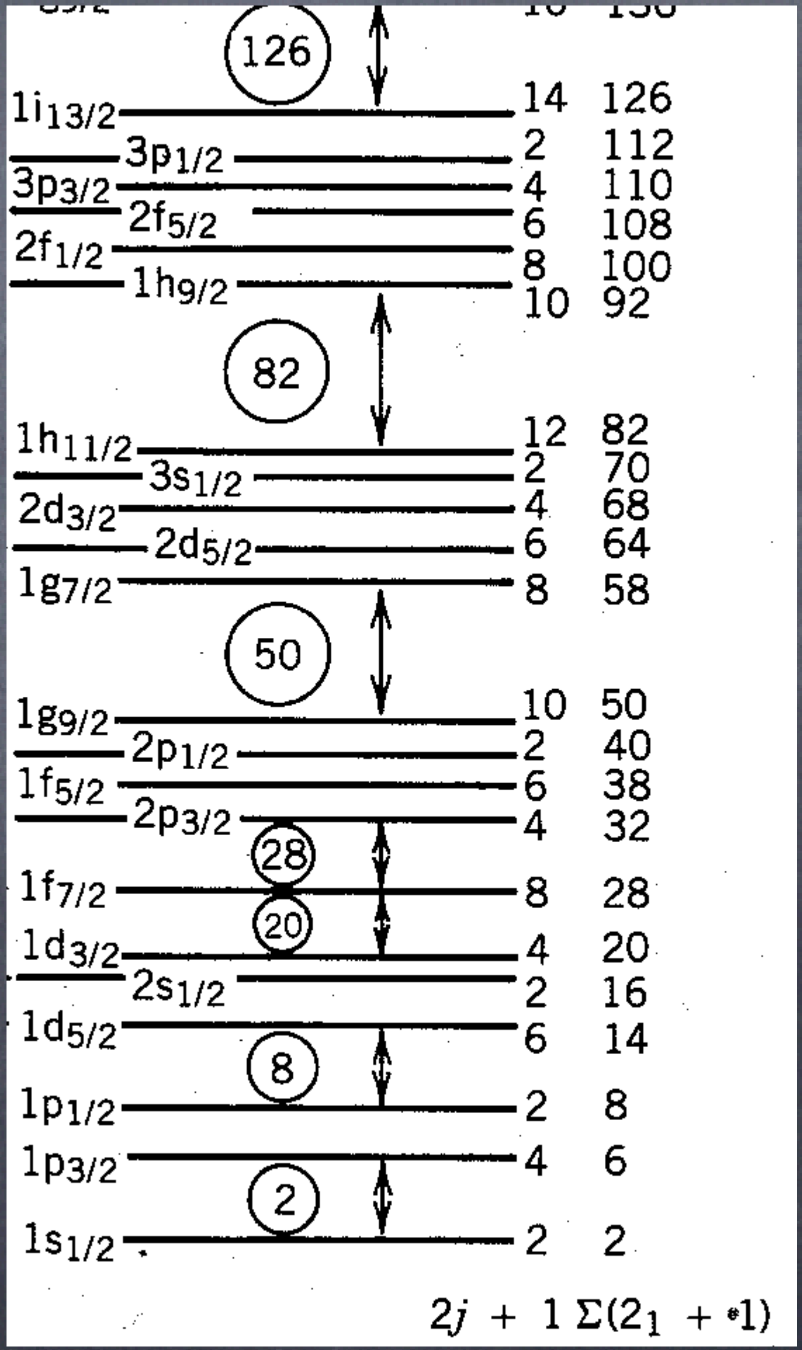


Excited states

- single-particle excitations
- Low-lying collective vibrations
- Giant resonances

Interpretation of ^{130}Sn (Z=50, N=80) spectrum





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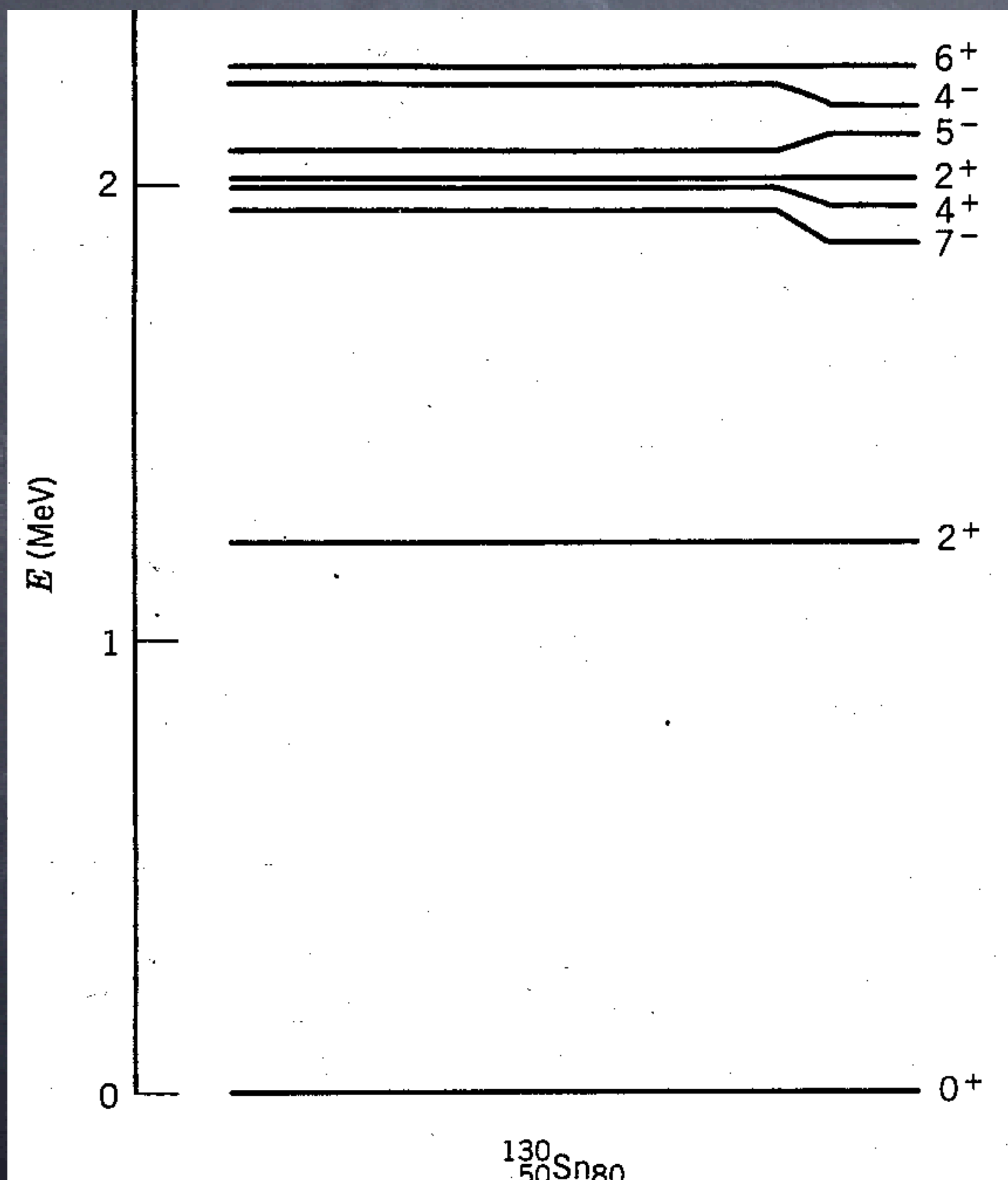

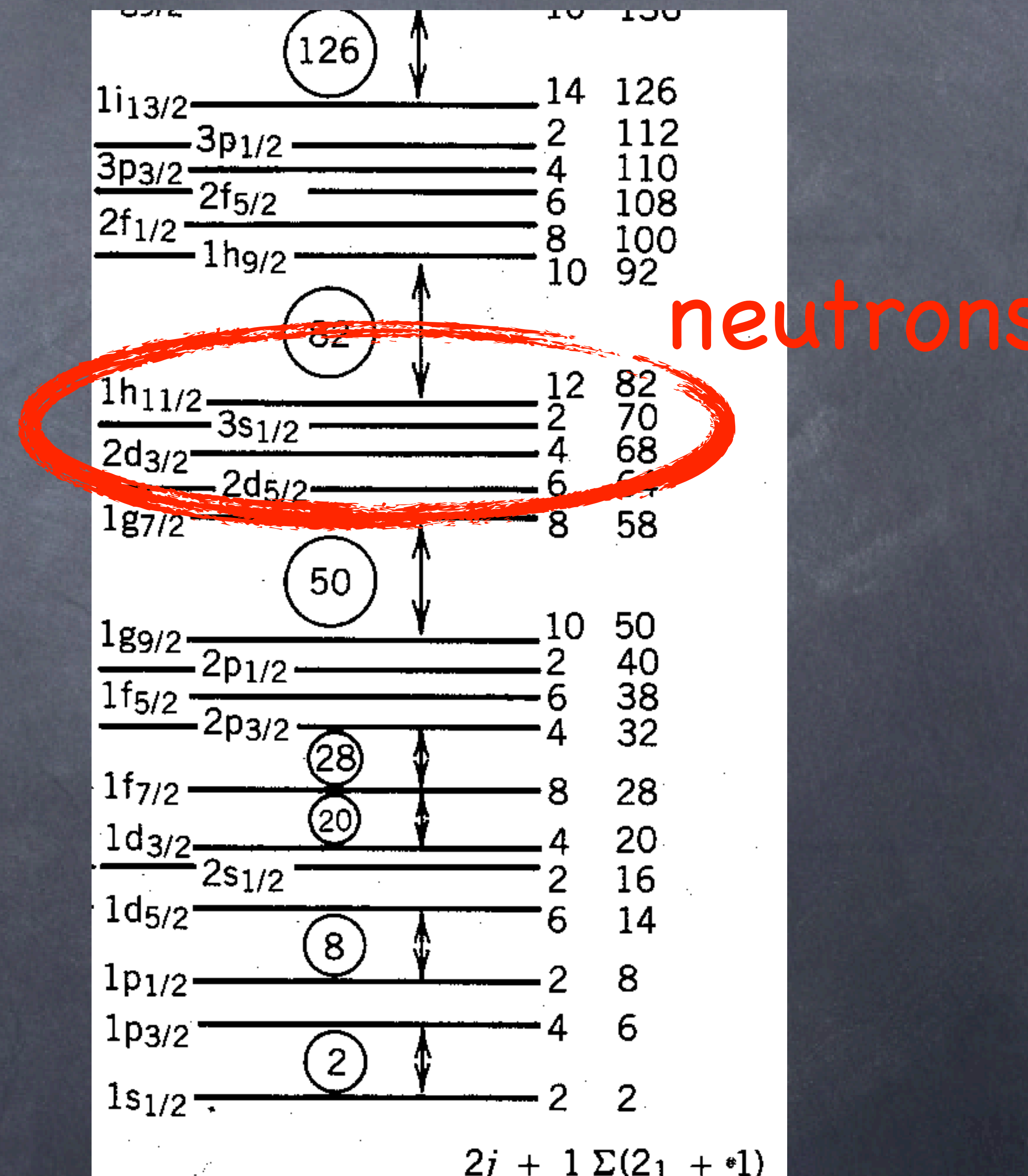

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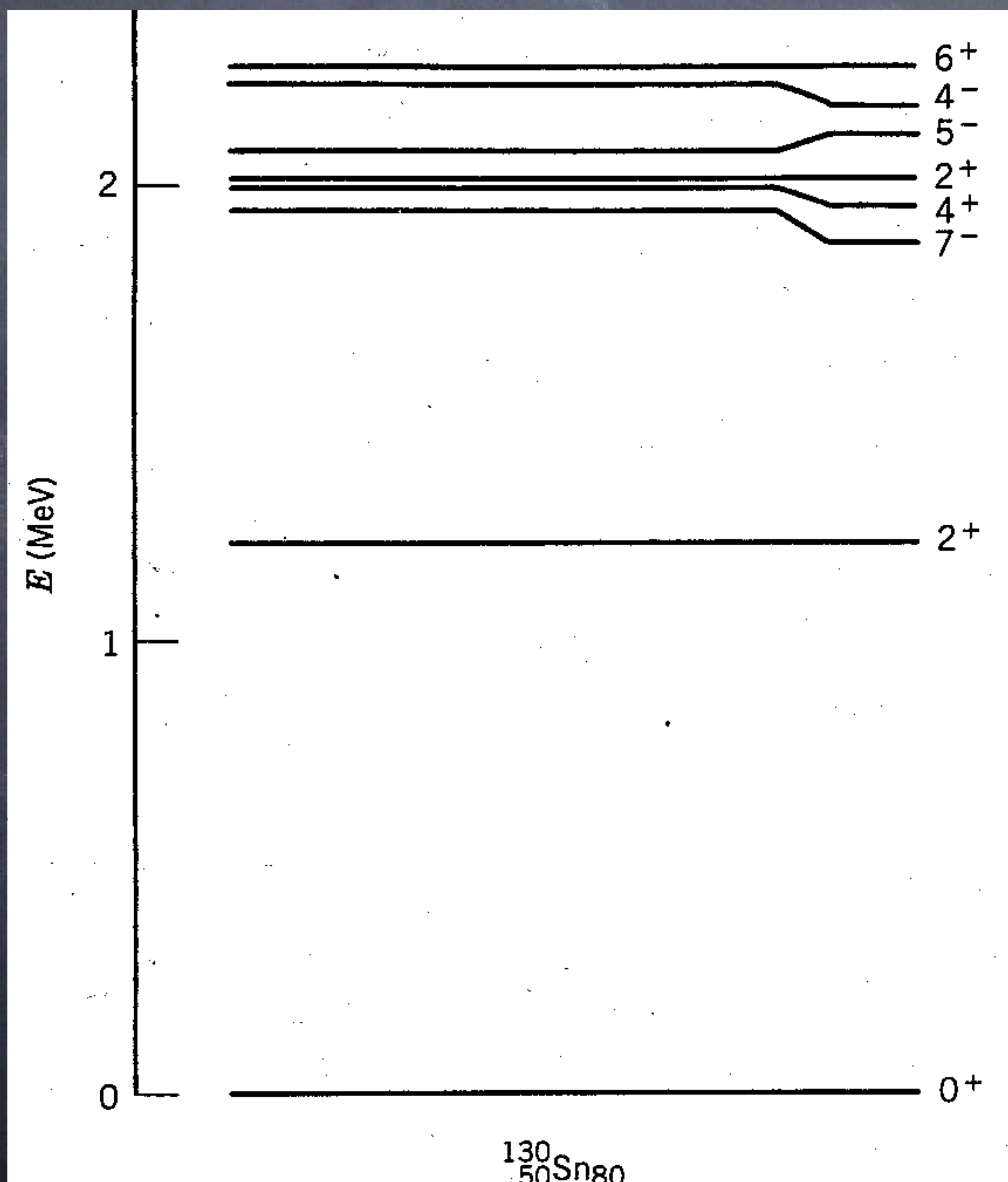

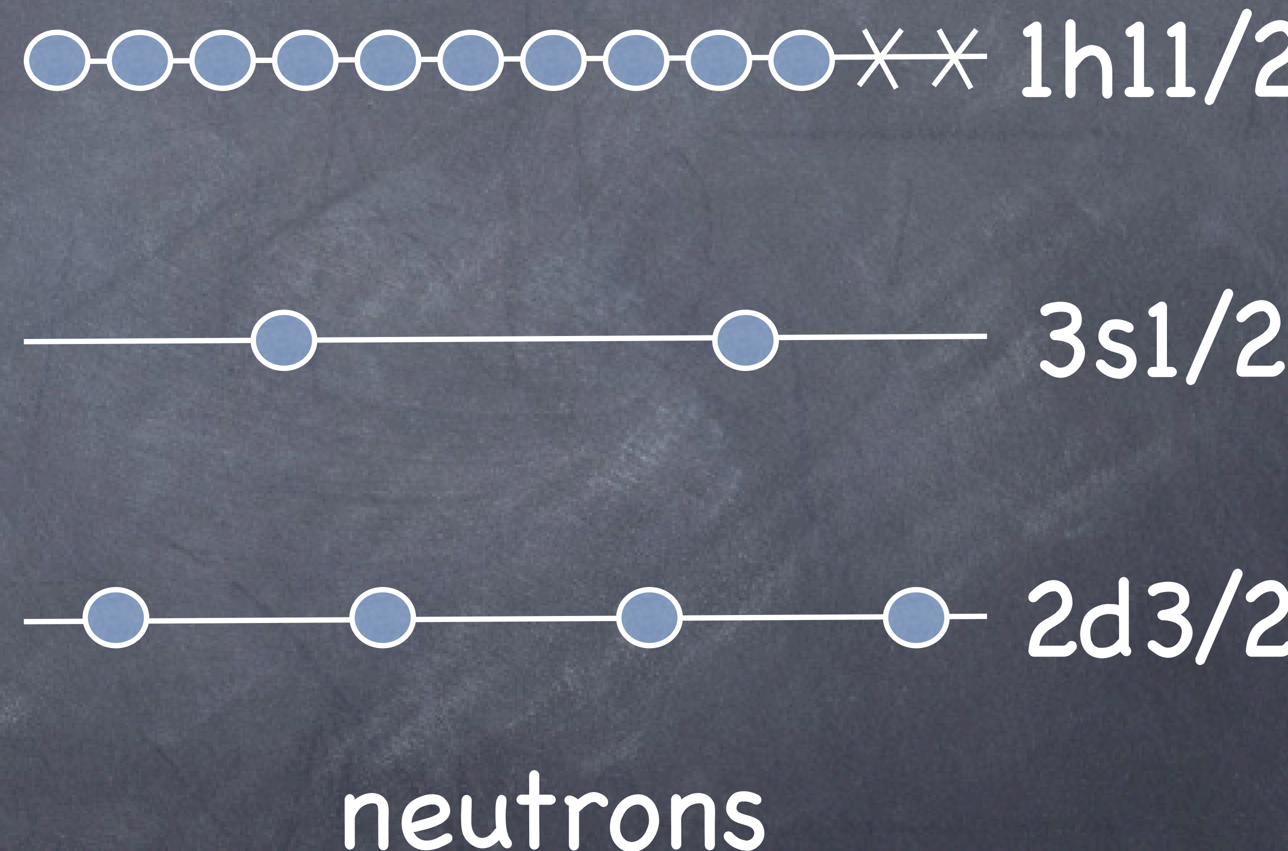

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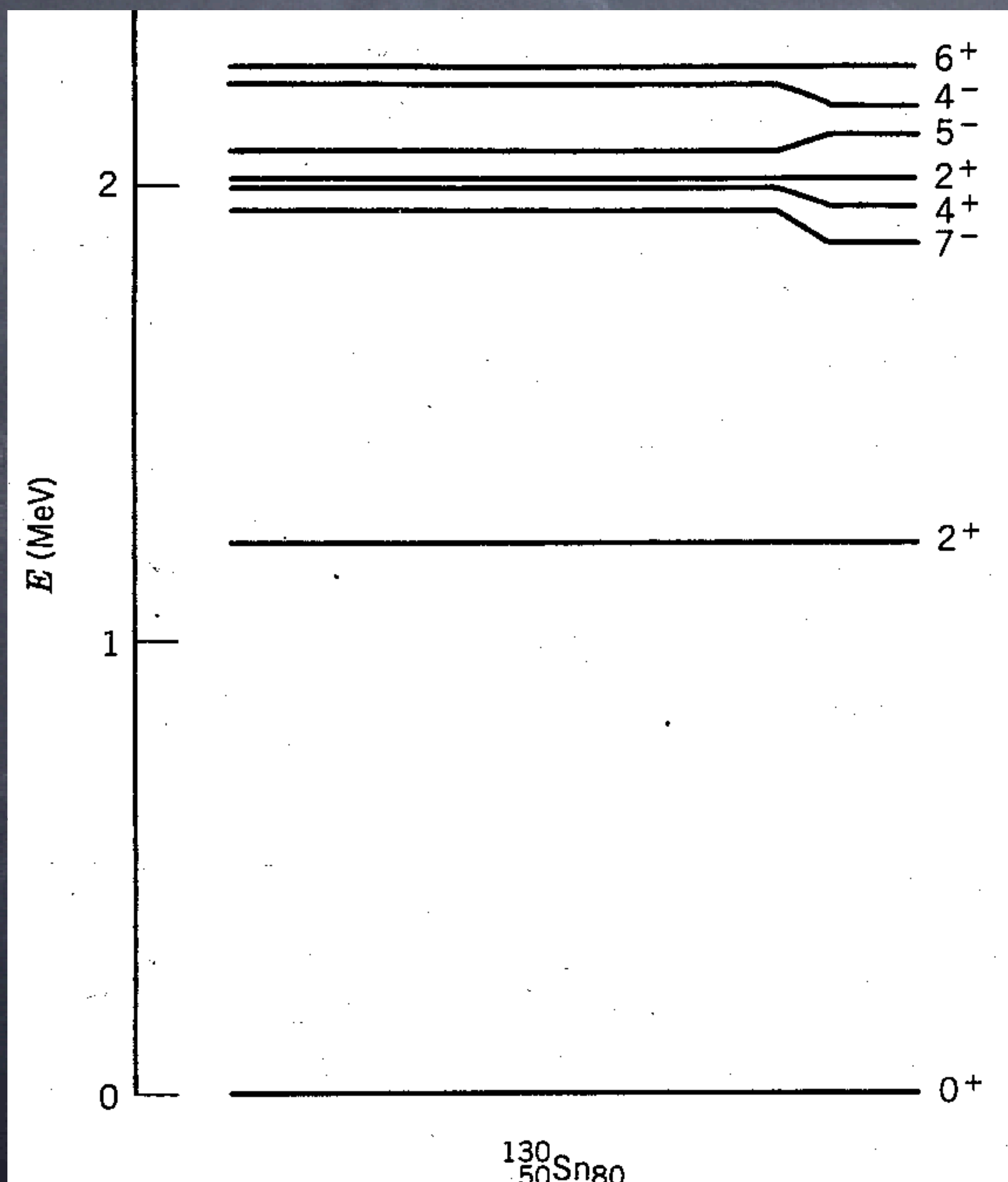

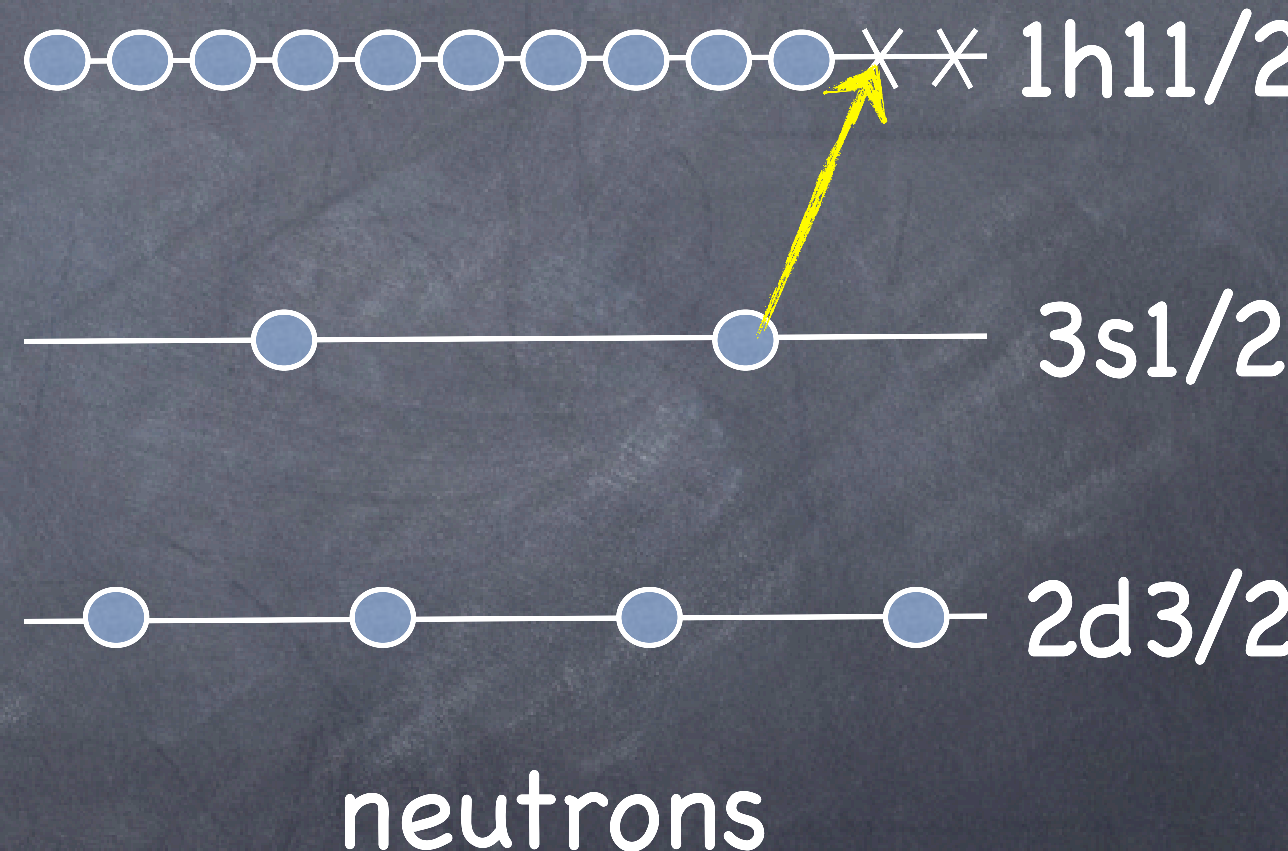

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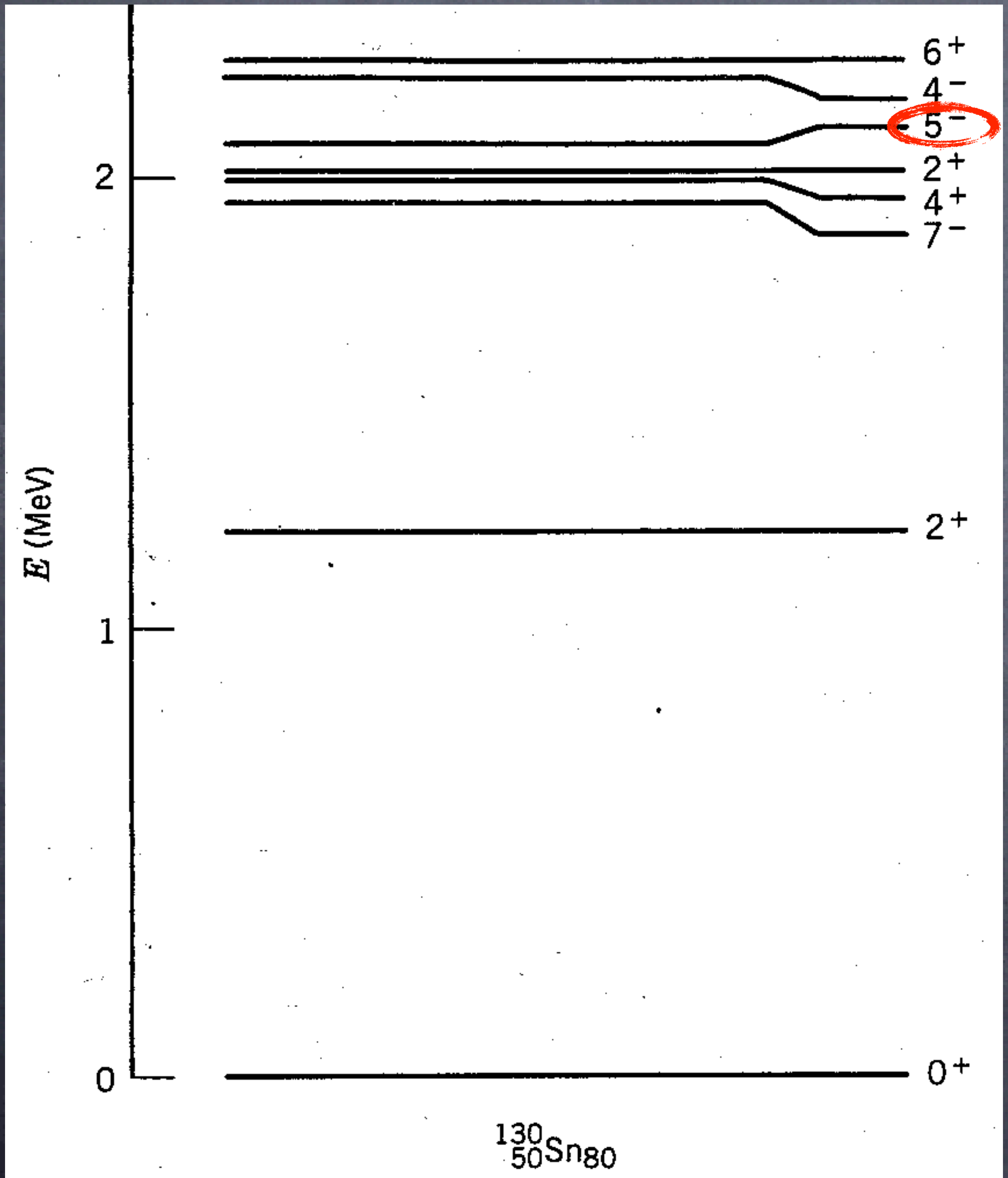

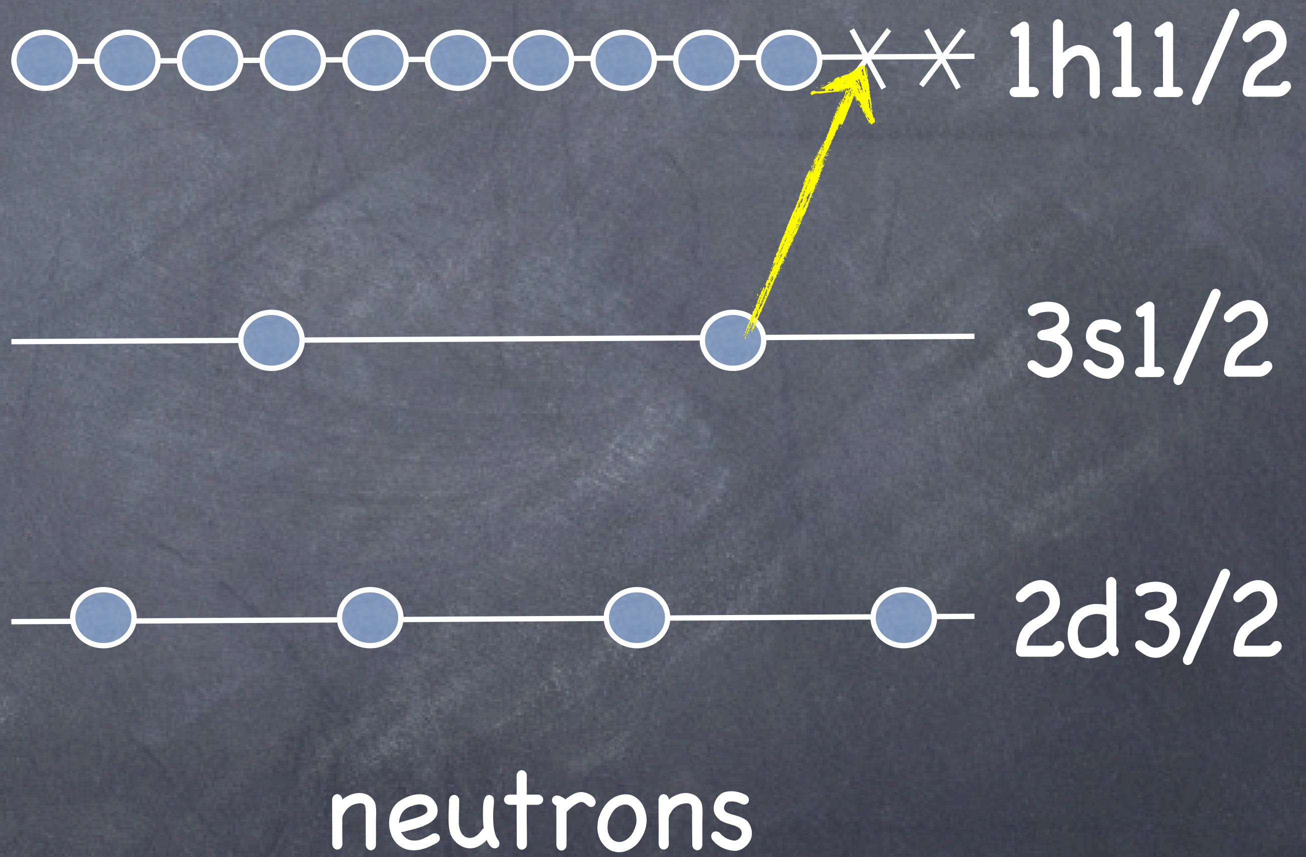

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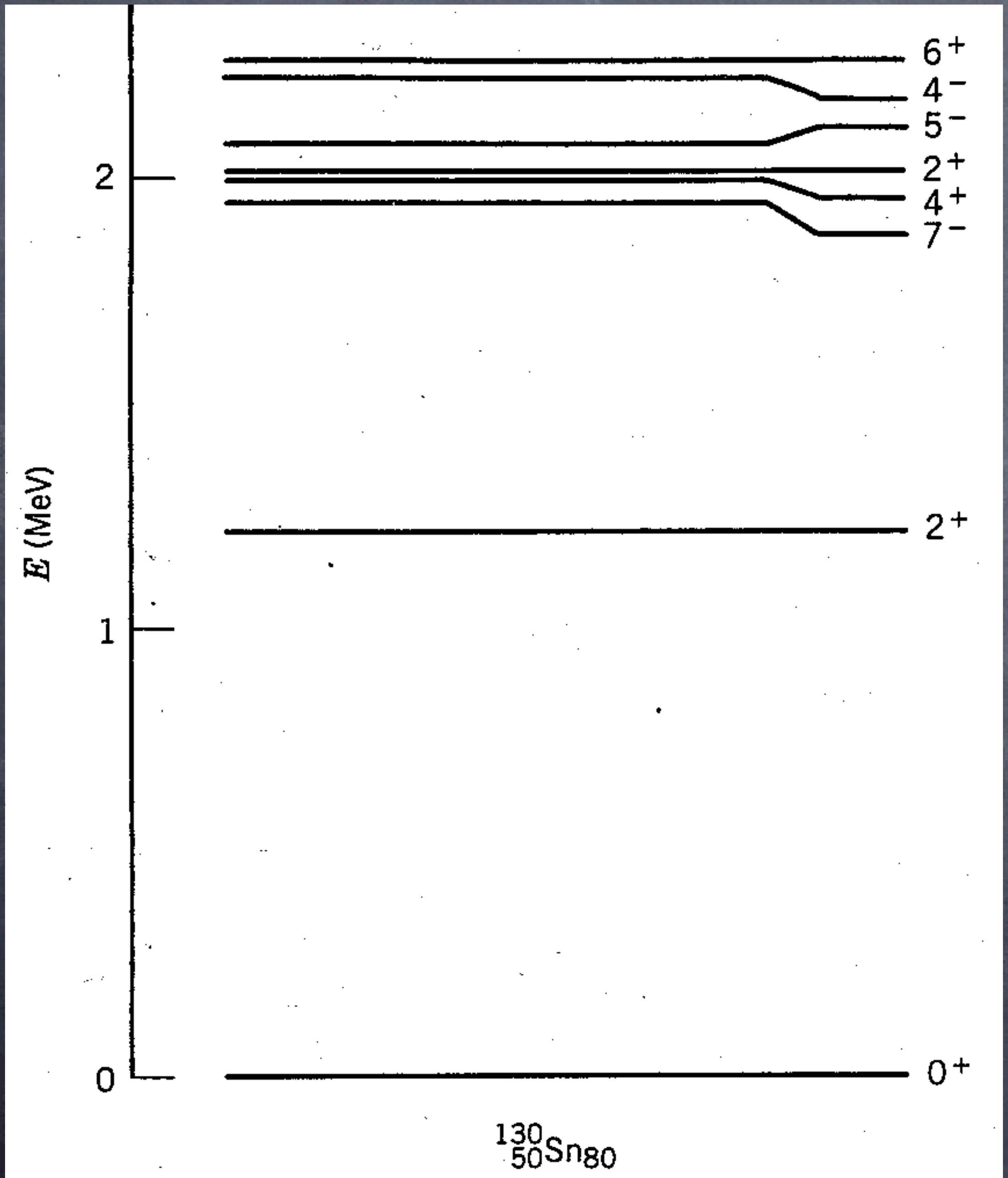

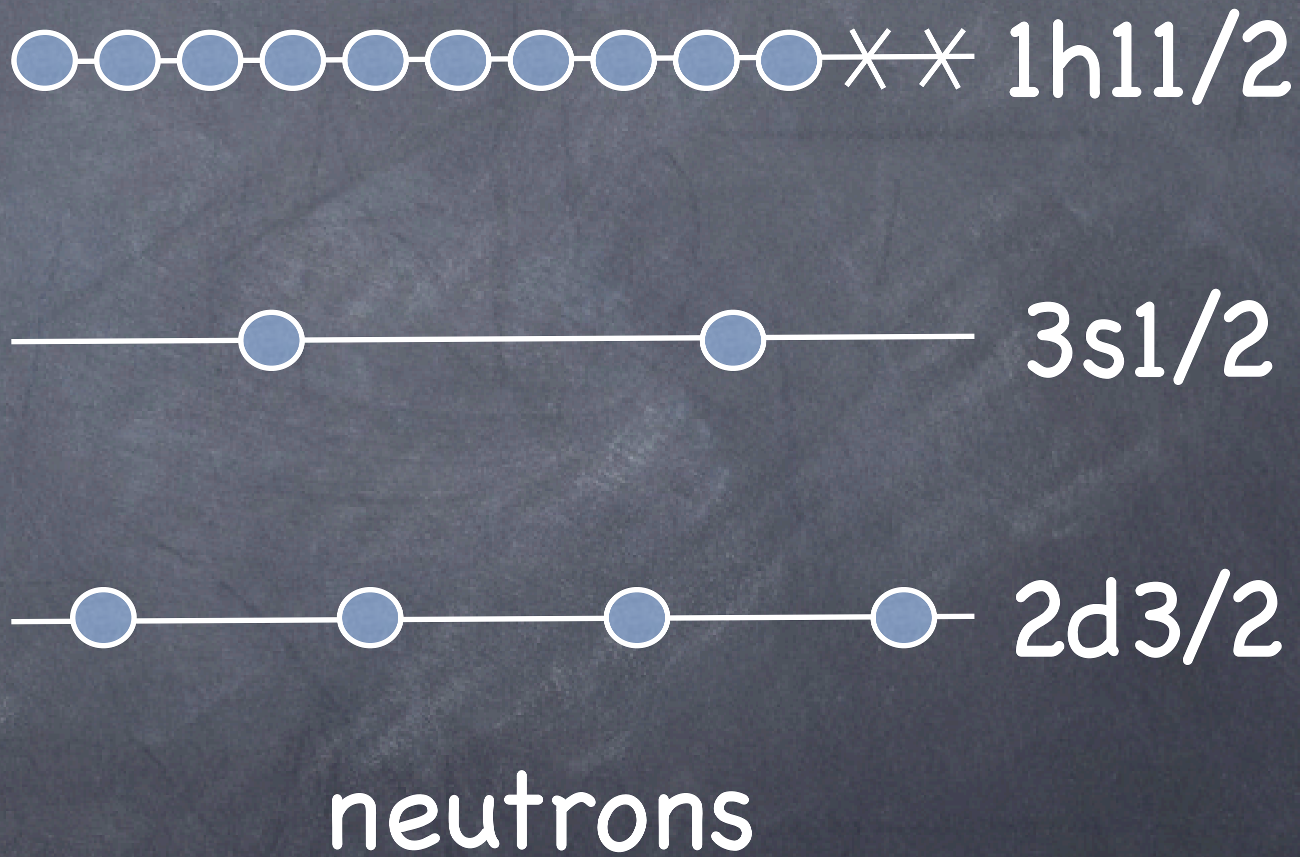

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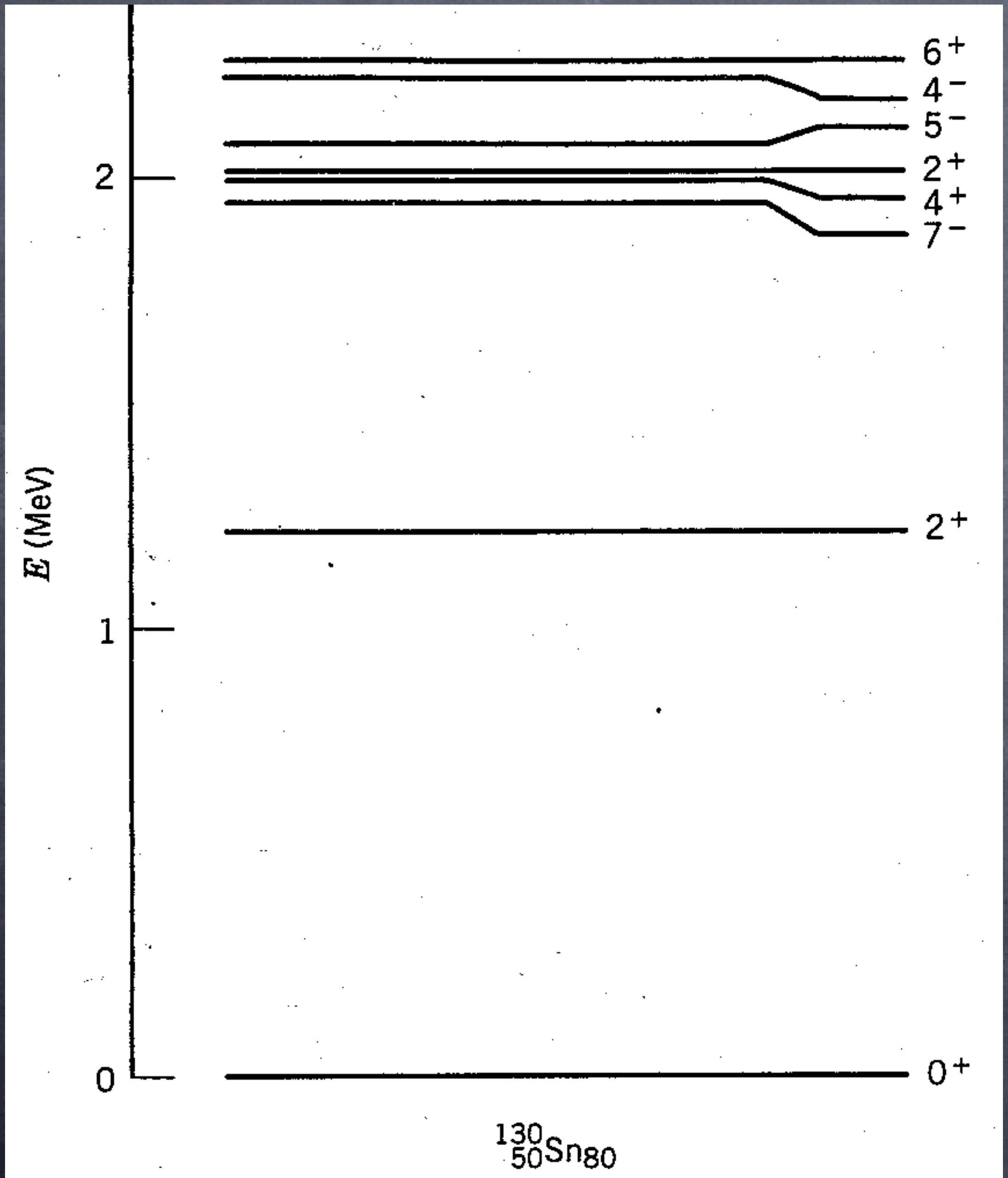

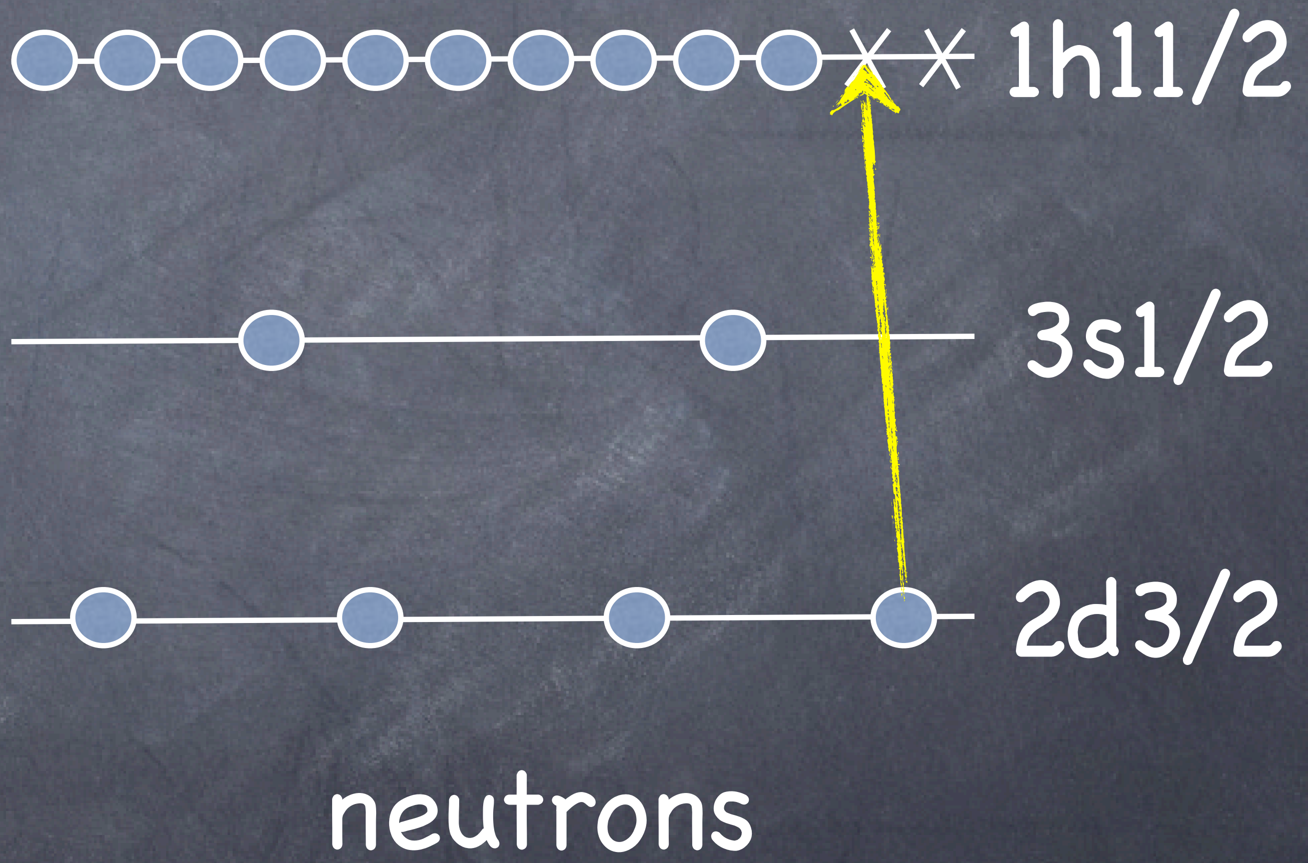

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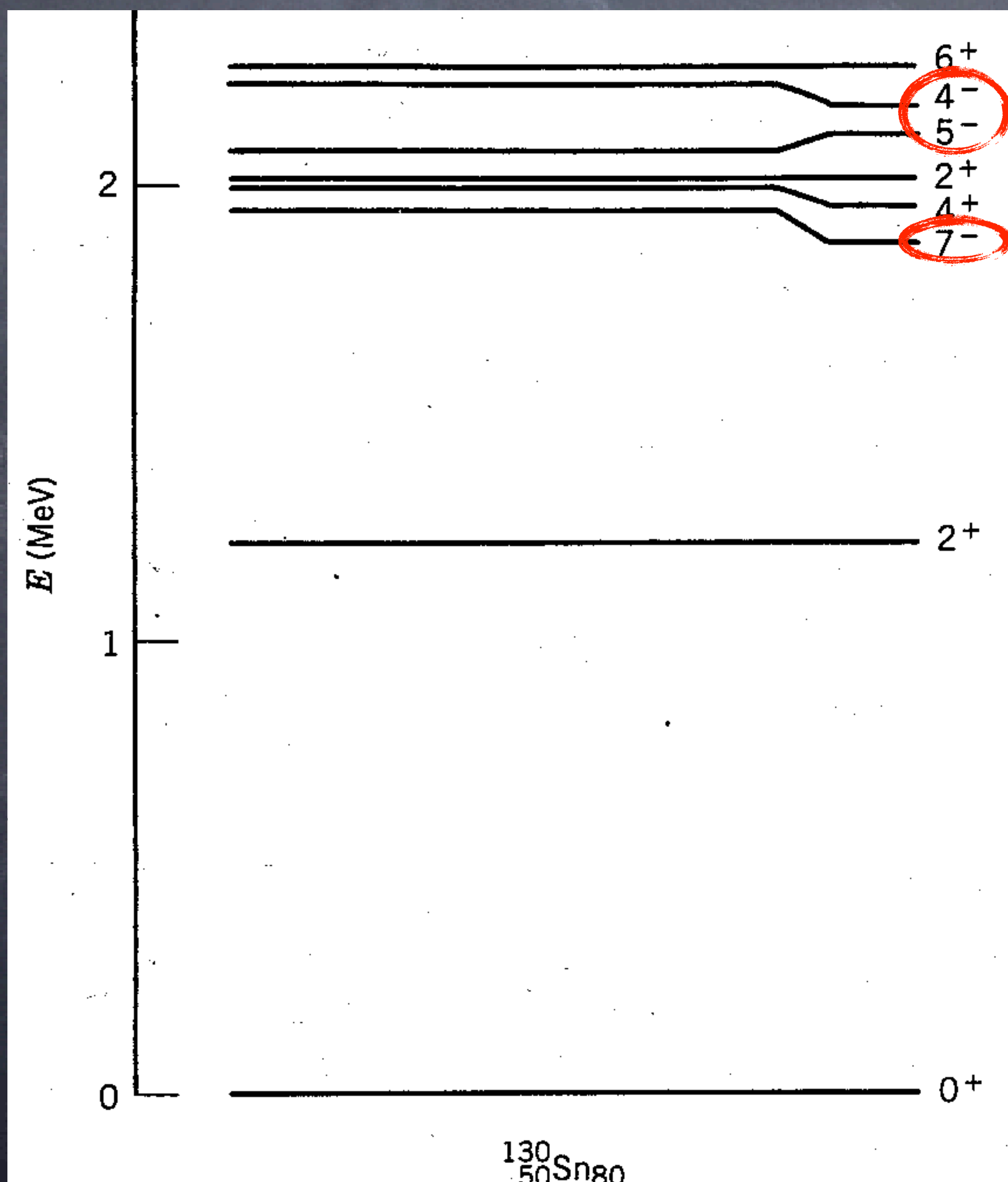

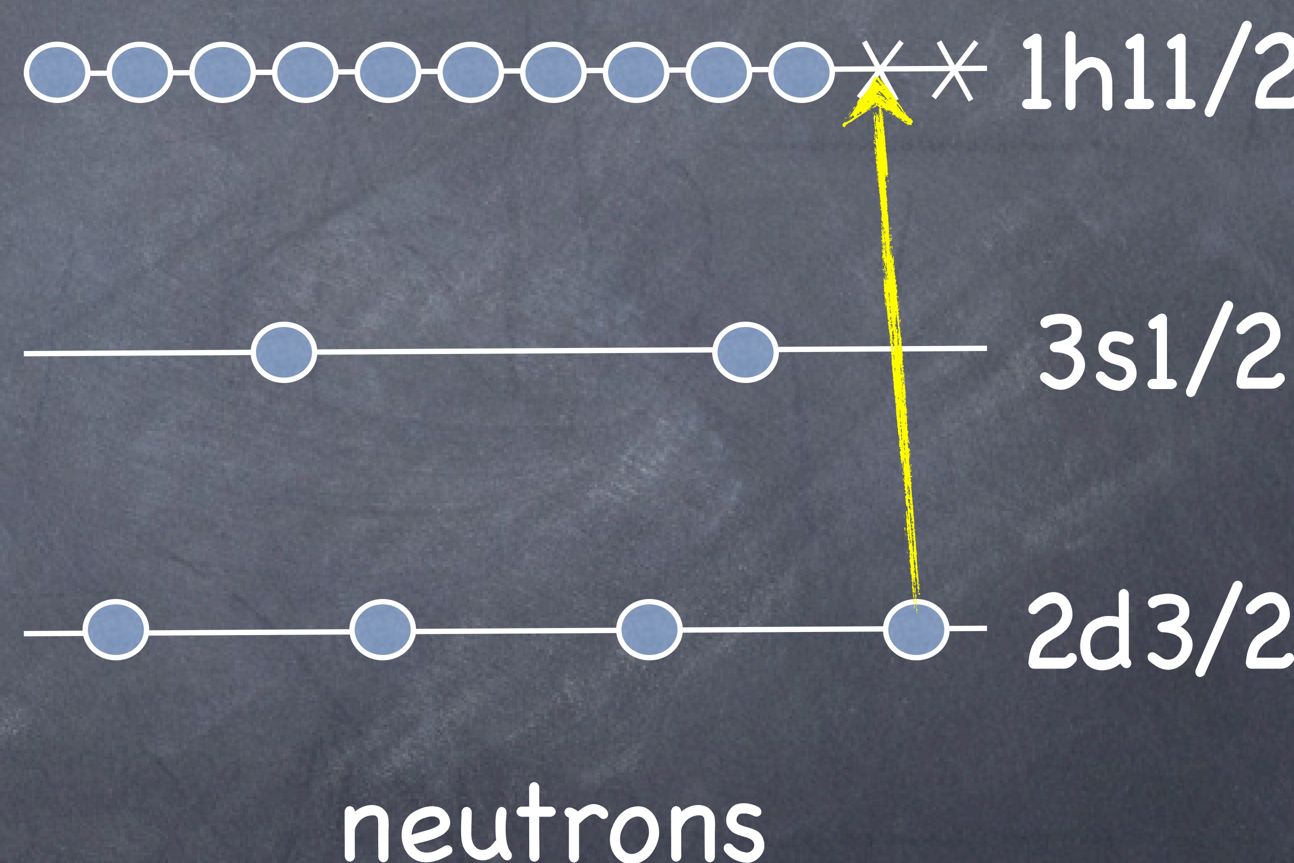

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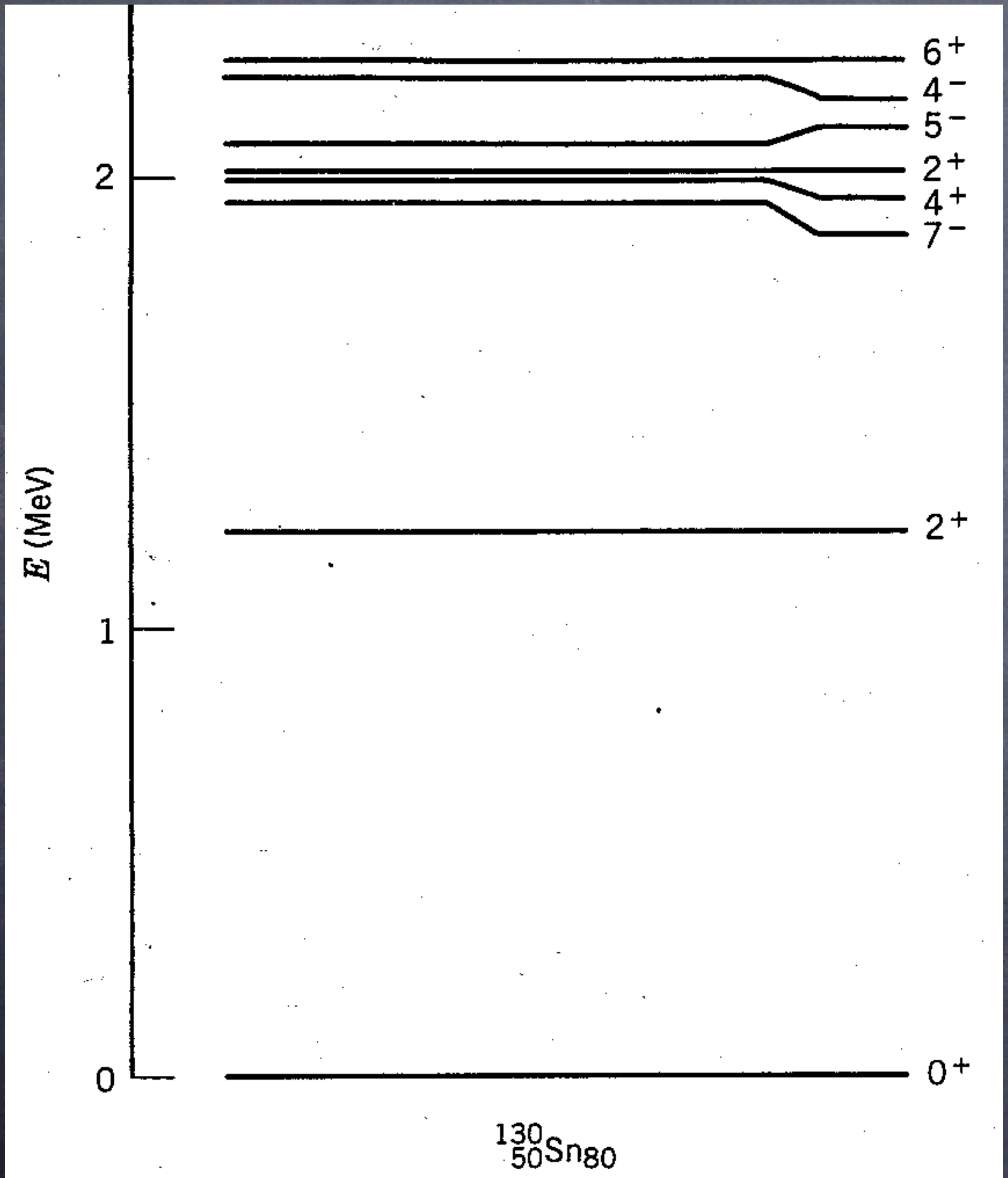

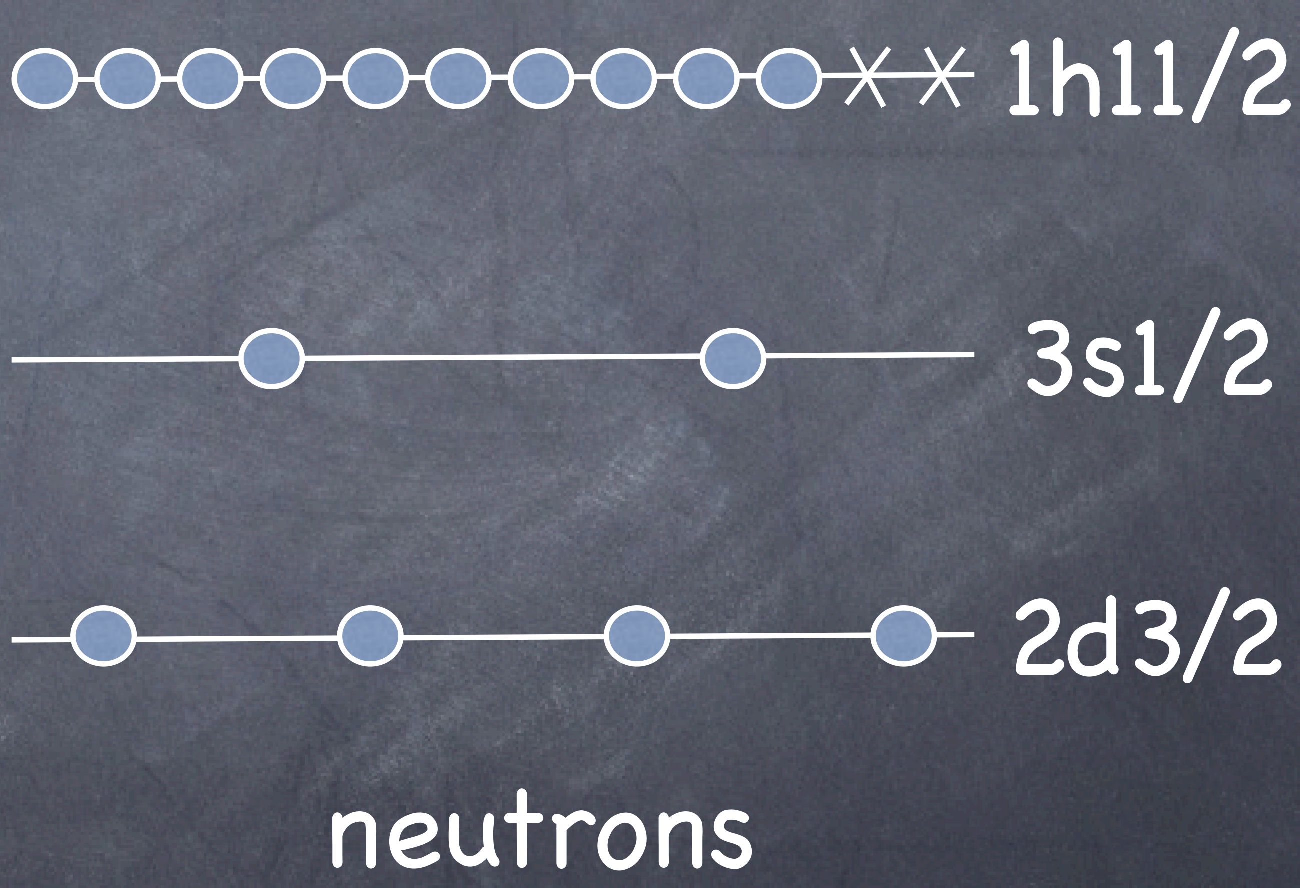

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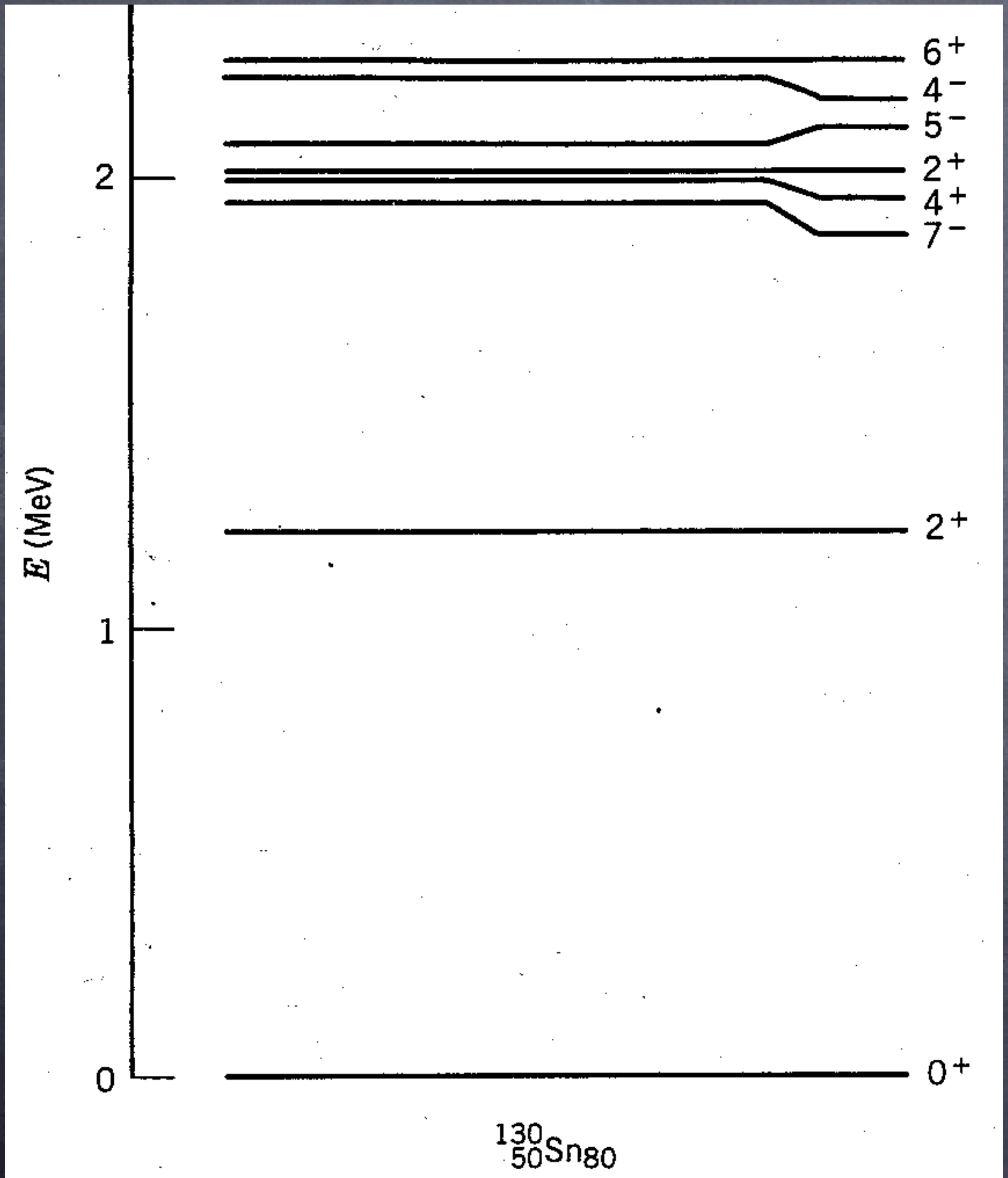

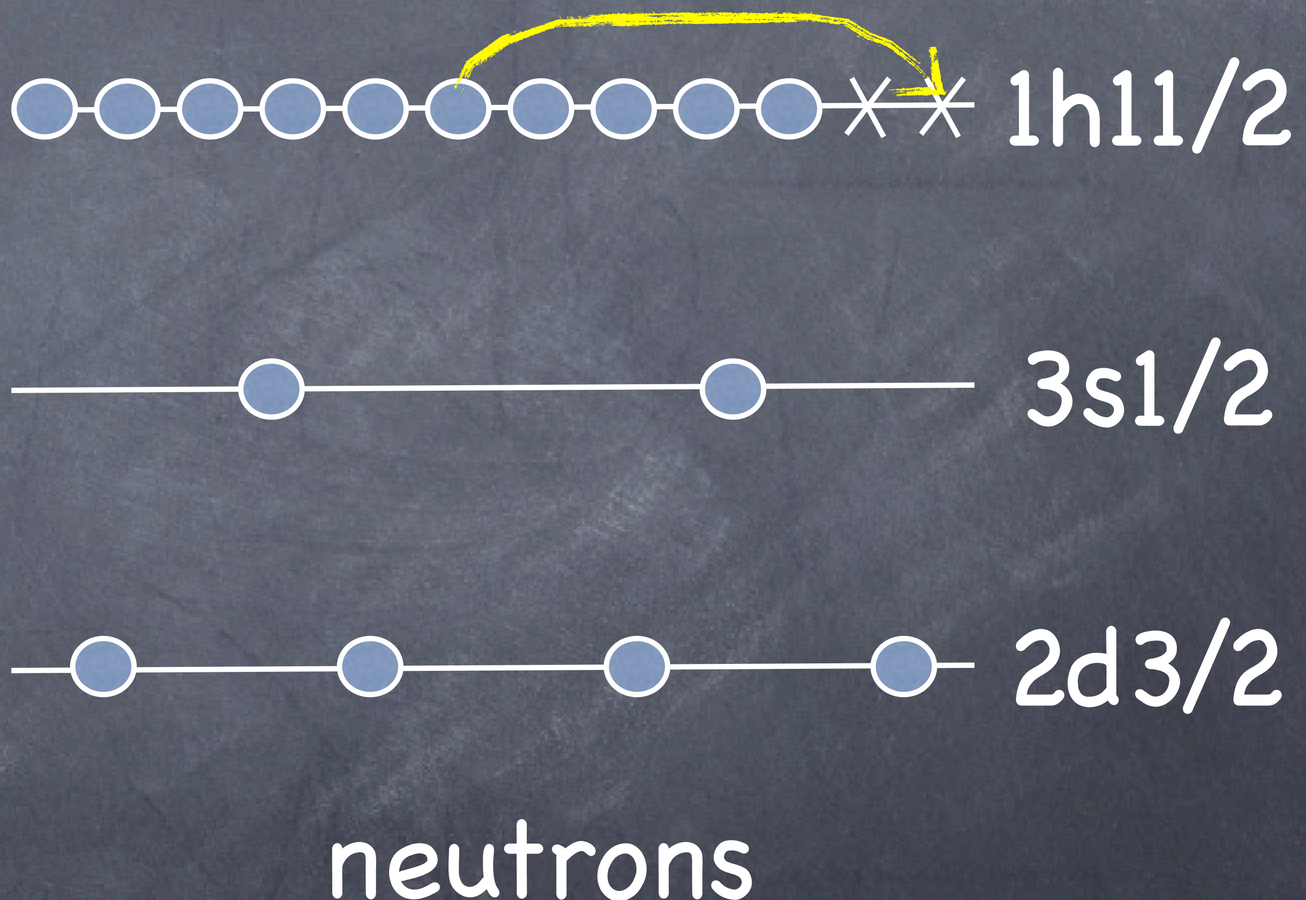

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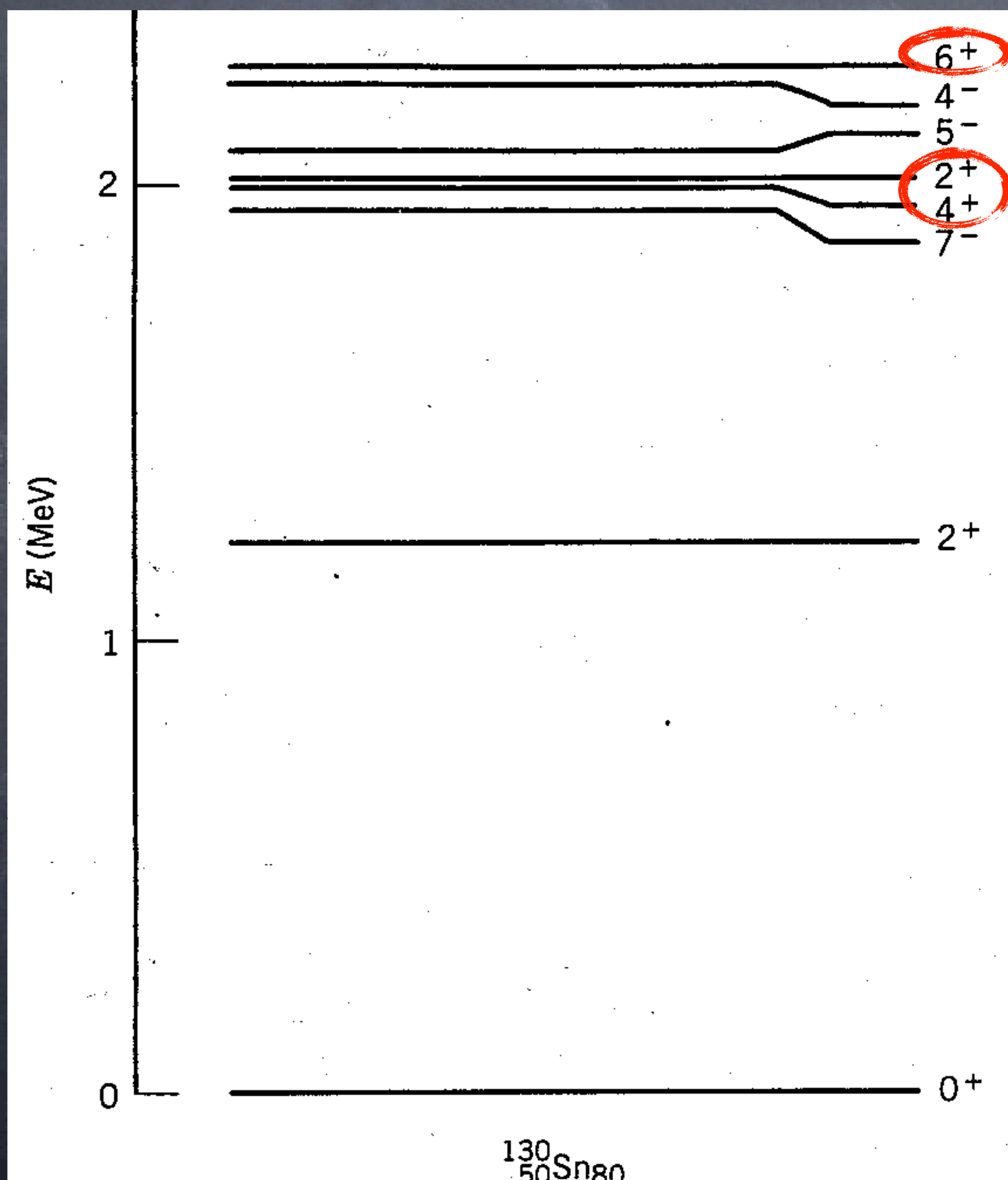

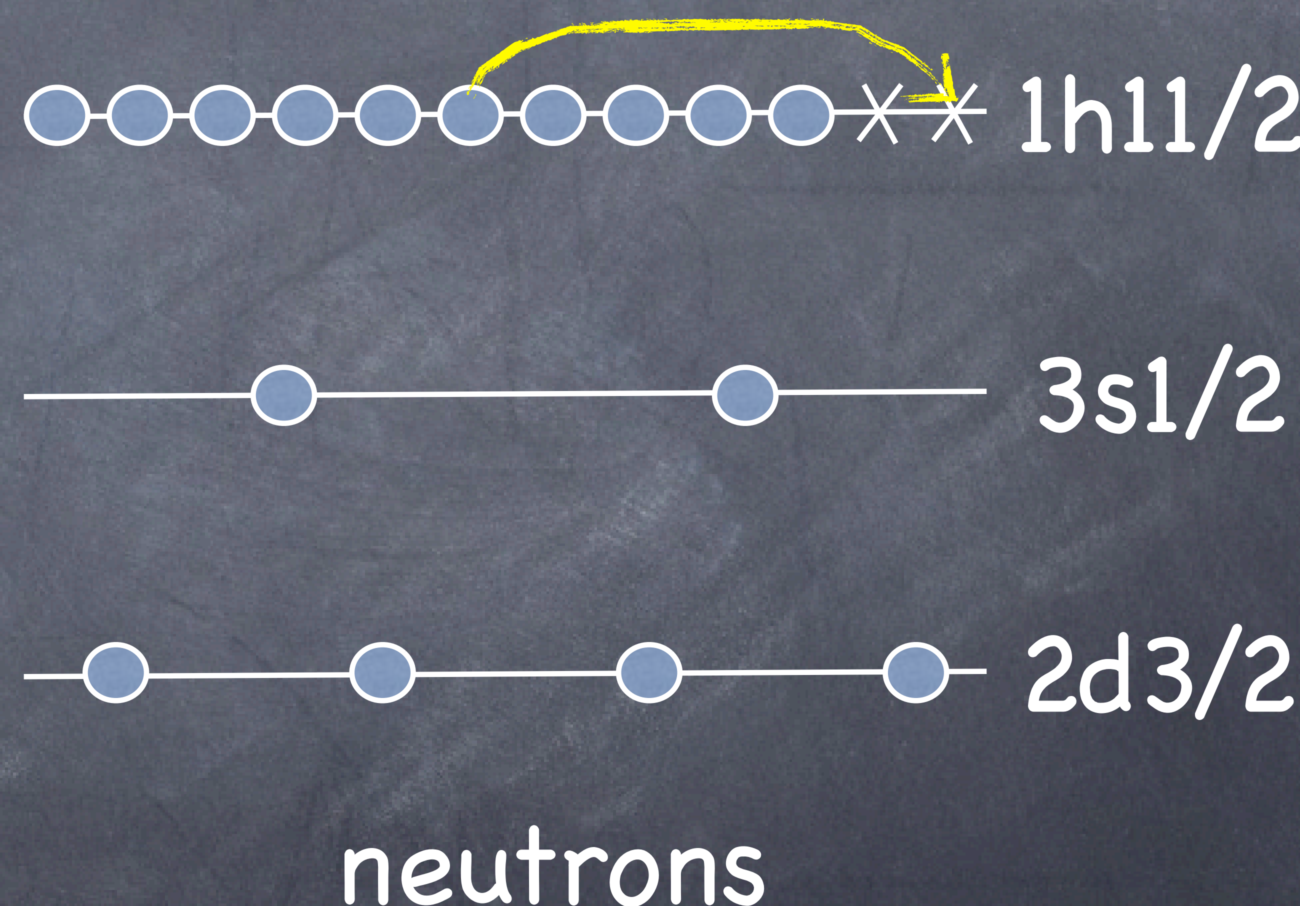

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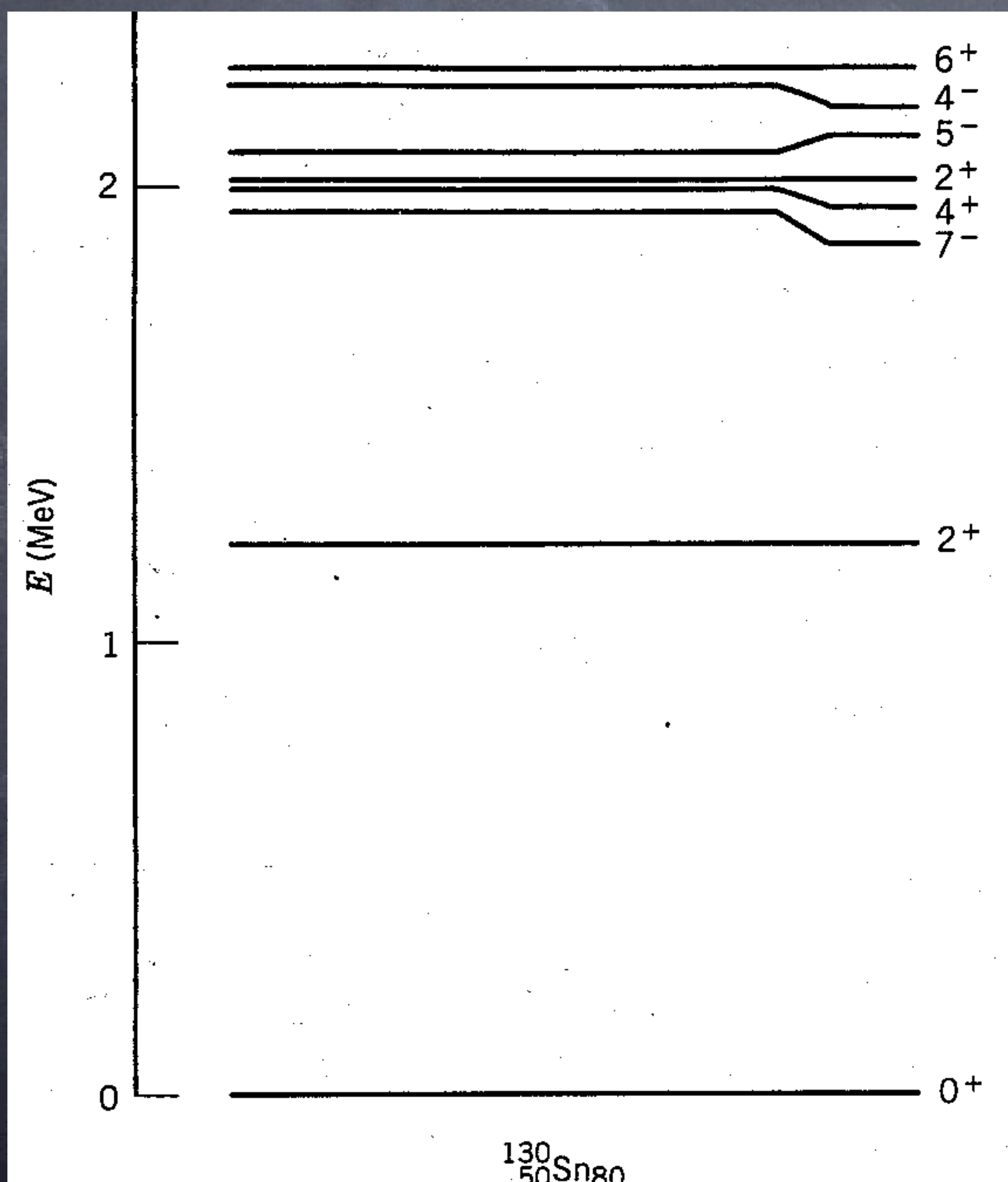

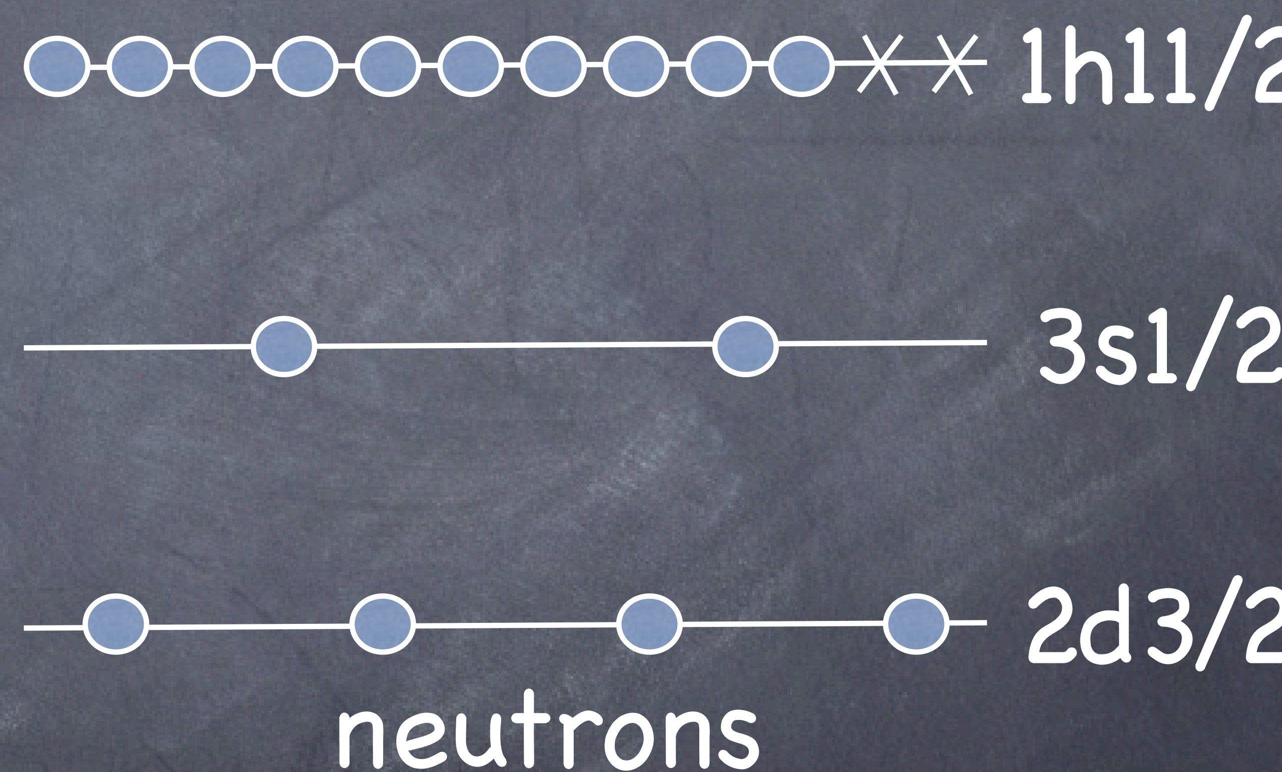

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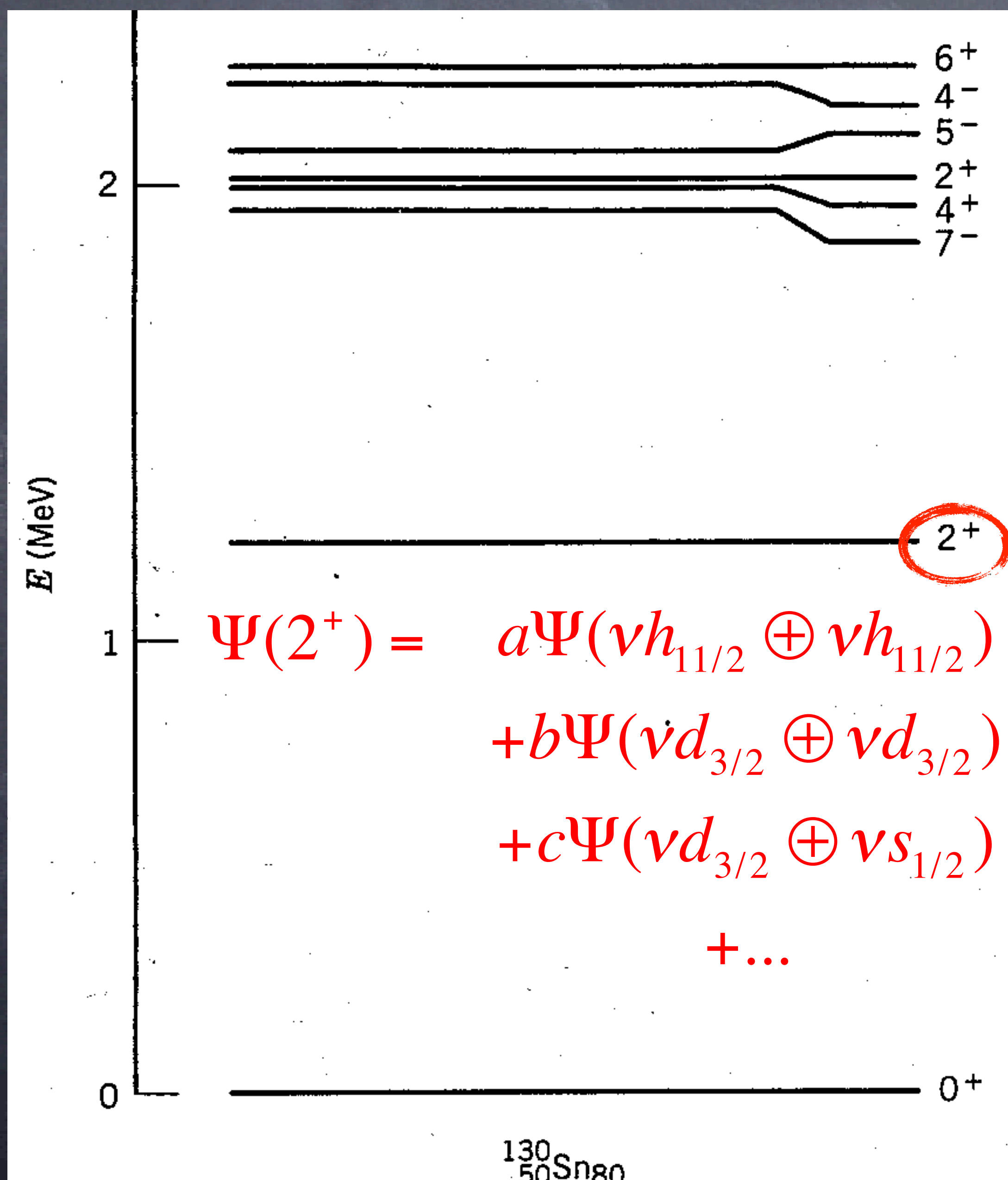
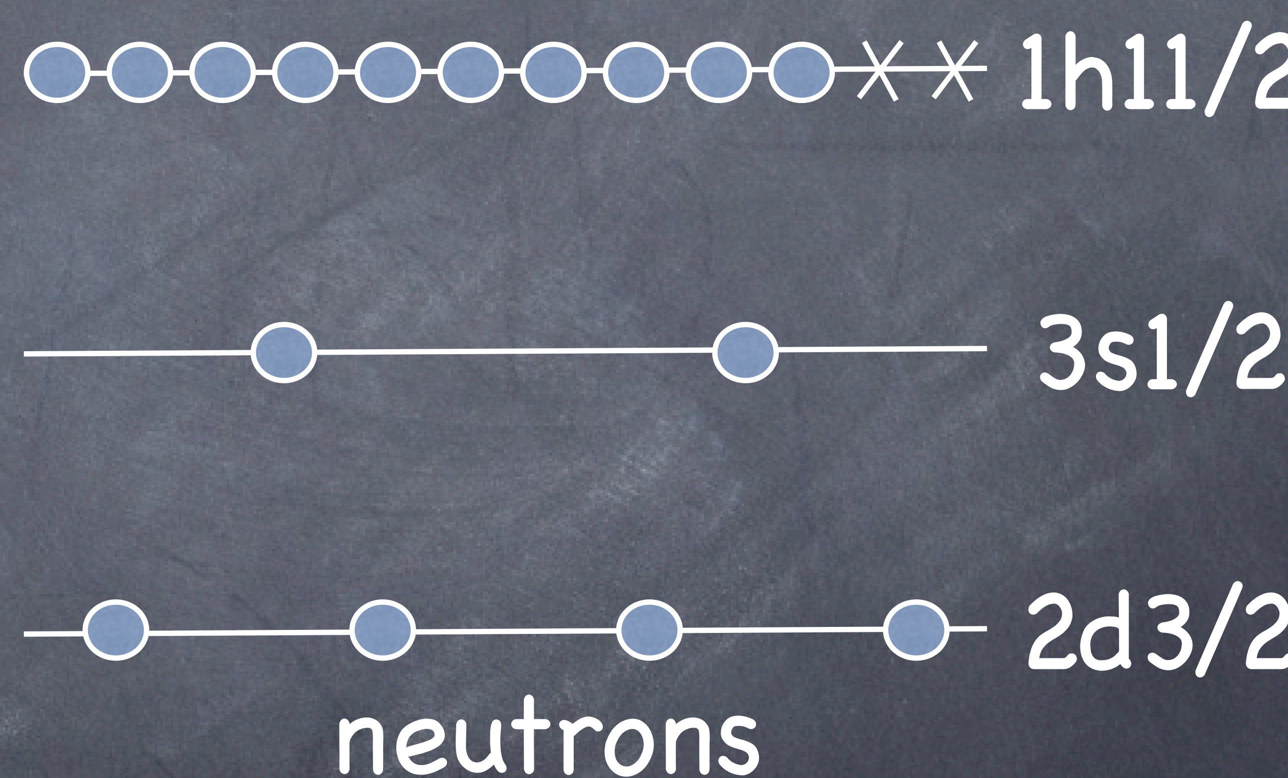


$$\Psi(2^+) = a\Psi(\nu h_{11/2} \oplus \nu h_{11/2}) + b\Psi(\nu d_{3/2} \oplus \nu d_{3/2}) + c\Psi(\nu d_{3/2} \oplus \nu s_{1/2}) + \ldots$$



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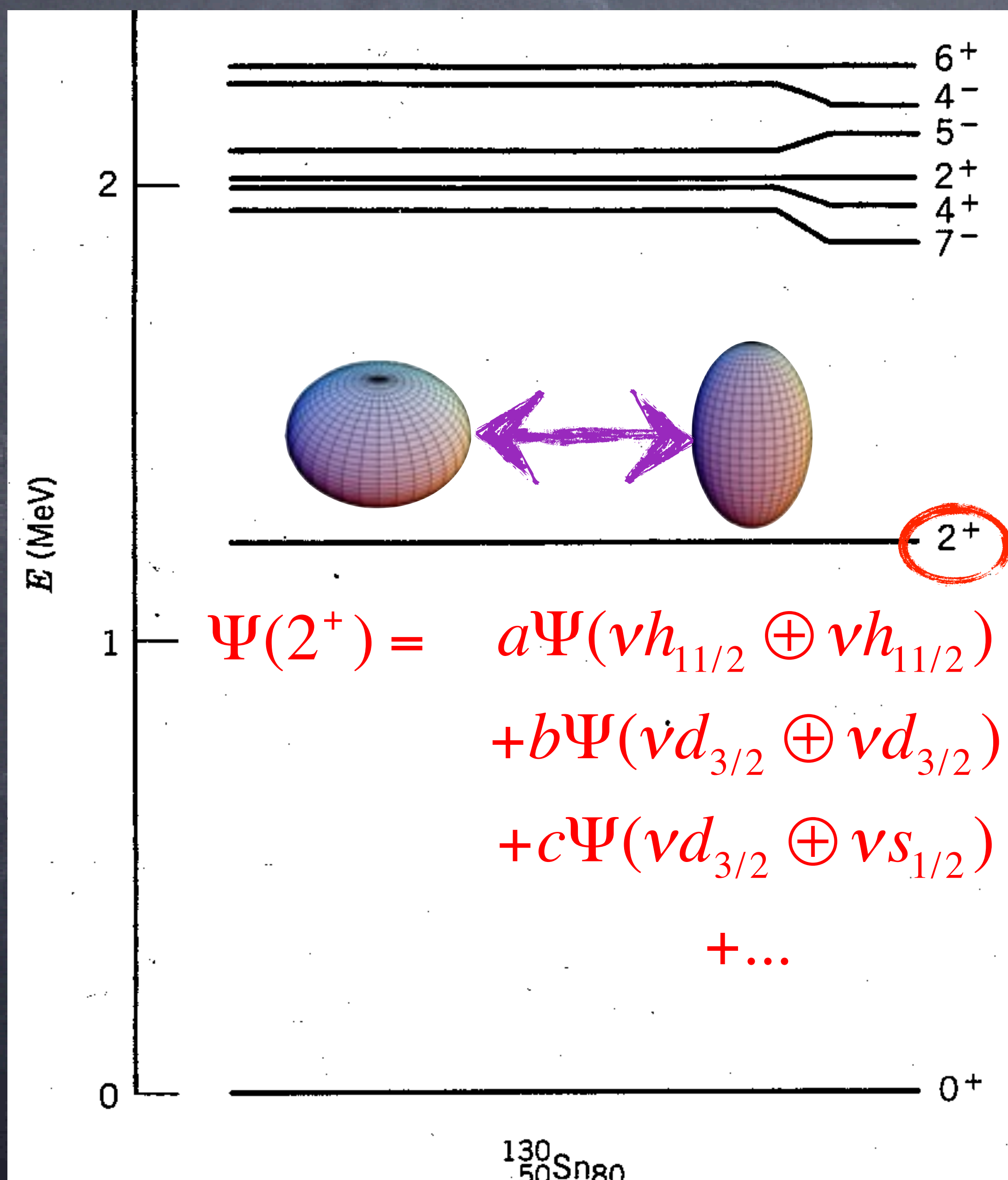

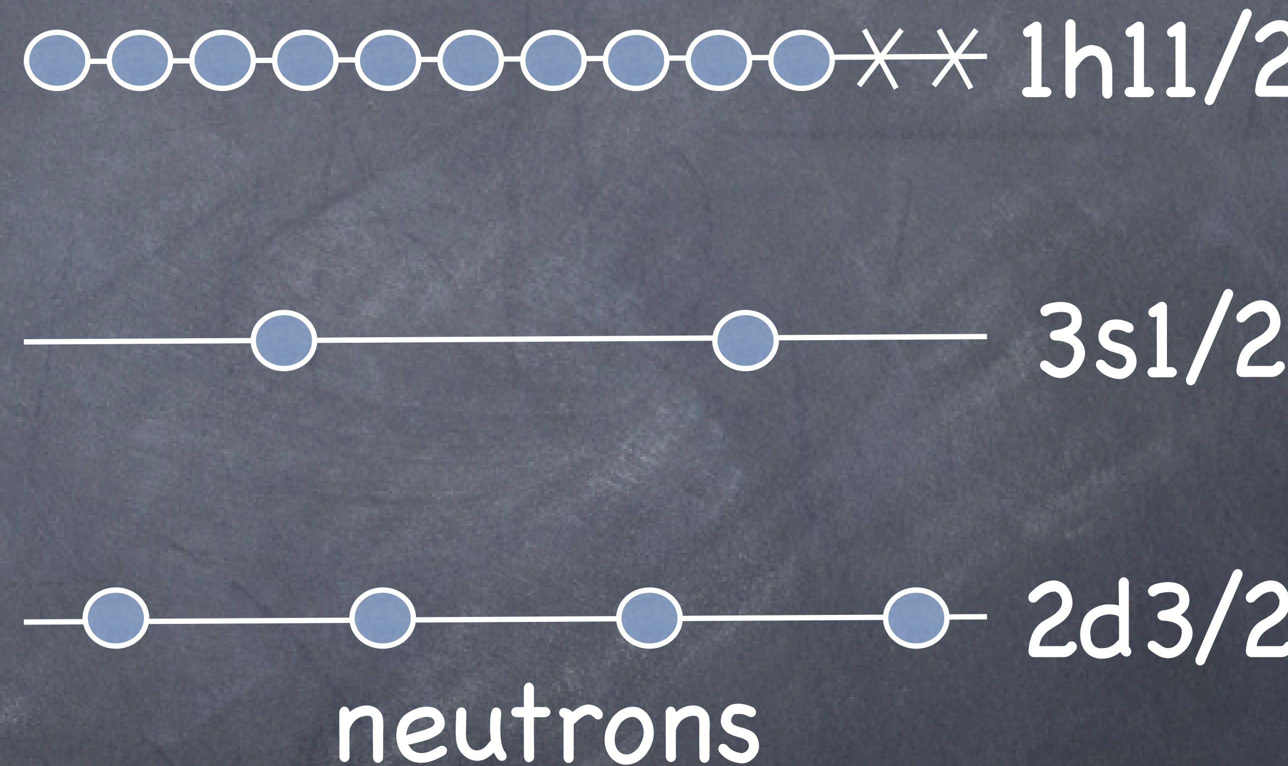

=> low-lying collective vibration ≠ HF eigenstate